

Theory of **SEAKEEPING**

by

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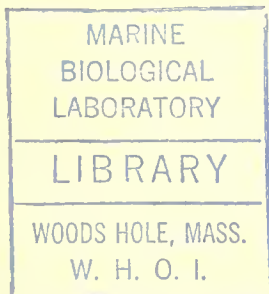


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velocity of translation of the facets of the wave form on which the predominating air pressures act. Both definitions would be identical if the waves were simple harmonic. The true wave structure consists, however, of smaller waves carried on the surface of the bigger ones, and the velocity of translation of small facets of wave form is therefore a complicated function of celerities of waves of all sizes.⁵

The value of the drag coefficient, $C_d = 0.0236$, agrees well with Motzfeld's model No. 4, and with the appearance of the waves shown on Fig. 11. It was derived on the basis of the mean wind speed of 23 fps at which a small c/V -ratio and large percentage of sharp-crested small waves can be expected.

2.6 Summary of C_d Data from Previous Sections.

It appears from the foregoing data that there is generally good agreement among the C_d -values obtained from pressure measurements in wind tunnel and flumes, and derived from the slope of the water surface. In a very mild case (Francis at 5 mps) and in very severe cases (Francis 12 mps and Johnson and Rice) the wave form can be shown to be analogous to Motzfeld's trochoidal and sharp-crested wind-tunnel models. However, for the cases of intermediate severity, the problem of defining the sea surface in a form significant for the drag has not been solved. Clearly λ/H -ratio is not the desired parameter. Its effect is practically discontinuous; a very slowly rising value of C_d with decreasing λ/H , and then a quick and drastic jump to a high value as sharp wave crests are developed. It appears that the C_d values are governed by a statistical parameter depending on the frequency of occurrence of sharp crests or possibly on the frequency of occurrence of steep wave slopes. The problem has to be treated therefore by statistical methods. These will be discussed later in Section 8 of this chapter.

It can be added here that in the wind-flume experiments rather strong wind velocities and short fetches were used, so that the waves were short (of the order of 1 ft) and the ratio c/V of wave celerity to wind speed was very small (less than $1/10$). The predominating waves in the actual sea have a c/V ratio of the order of 0.86 (Neumann 1953, p. 21). The wind-flume data, therefore, although valuable material for the study of various relationships, do not represent ocean conditions directly. The method of analyzing such data thus becomes particularly important. Analyses in the past were generally inadequate because of limitation to overall drag of the water surface, neglect of wave irregularity, and the indeterminateness of the wind velocity to which the data are referred. An important observation in connection with wind-flume tests is that the wave system is very irregular from the outset. The irregularity was commented upon by Francis (1951), and is demonstrated quantitatively by Johnson and Rice (1952), who present a number of graphs of the statistical distribution of wave heights and periods. These are reproduced here in Fig. 12.

⁵ It was derived statistically by Longuet-Higgins (1955, 1957).

2.7 Estimation of Tangential Drag from Wind-Velocity Gradient.⁶ The method of estimating the drag coefficient of a sea surface by measuring the velocity gradient in wind is based on the theory of turbulent boundary layer at a rough surface. The tangential resistance of such a surface causes momentum loss in the immediately adjacent layers of air. The turbulent movements of air particles cause a momentum transfer from one layer of air to another, and, as a result of this, the air velocity diminishes gradually from the velocity of the undisturbed flow V at a large distance from the rough surface to a velocity $u < V$ as the surface is approached. The air velocity u is therefore a function of the distance z from the plate; i.e., $u = u(z)$. The plot of velocity u versus height z has the general shape shown in Fig. 13. The tangential drag of a surface is equal to the shear stress in the air layers in close proximity to the surface and is expressed in the general form as

$$\tau = A(du/dz)_{z=0} \quad (23)$$

where τ is the shearing (or frictional) force per unit area and the coefficient A is yet to be defined. In the most common usage a coefficient of hydrodynamic force is defined in terms of $\rho V^2/2$, as for example in equation (14). In aerodynamic usage in Great Britain, however, it became customary to express it in terms of ρV^2 . This usage has been generally adopted in the field of oceanography, and the tangential force coefficient is written as

$$\gamma = \tau/\rho V^2$$

or preferably

$$\gamma = \tau/\rho u_z^2 \quad (24)$$

In the foregoing expression u denotes the air velocity as measured at a certain specific height z' . It follows then that coefficients C_d^* and γ are related by

$$C_d^* = 2\gamma$$

Attention should be called to the fact that C_d^* represents the total drag coefficient; i.e., $C_d + C_r$ in the notation used in preceding paragraphs.

In the fields of aerodynamics and of hydrodynamics (as applied to ships) the distance z over which $u(z)$ is variable is generally small and it is easy to measure the fluid velocity at a distance from the body where $u(z) = V$. In meteorology and oceanography it is necessary to consider the wind which has blown over a vast distance, and the height z over which $u(z)$ is appreciably variable is so large that it is impossible to measure velocities in the region where $u(z) = \text{const}$, except by means of pilot balloons. It becomes necessary, therefore, to establish the form of function $u(z)$, as well as certain conventions as to the height at which u should be measured.

These conventions have been only very loosely defined. G. I. Taylor (1915, 1916) used the data of pilot-balloon observations over Salisbury Plain in England.

⁶ For a more complete treatment of this subject the reader is referred to Ursell (M).



Fig. 11 Two views of downward end of pond taken at a wind speed of 17 m/sec before and after addition of detergent to the surface. The marker pole is graduated at 1-ft levels (from Van Dorn, 1953)

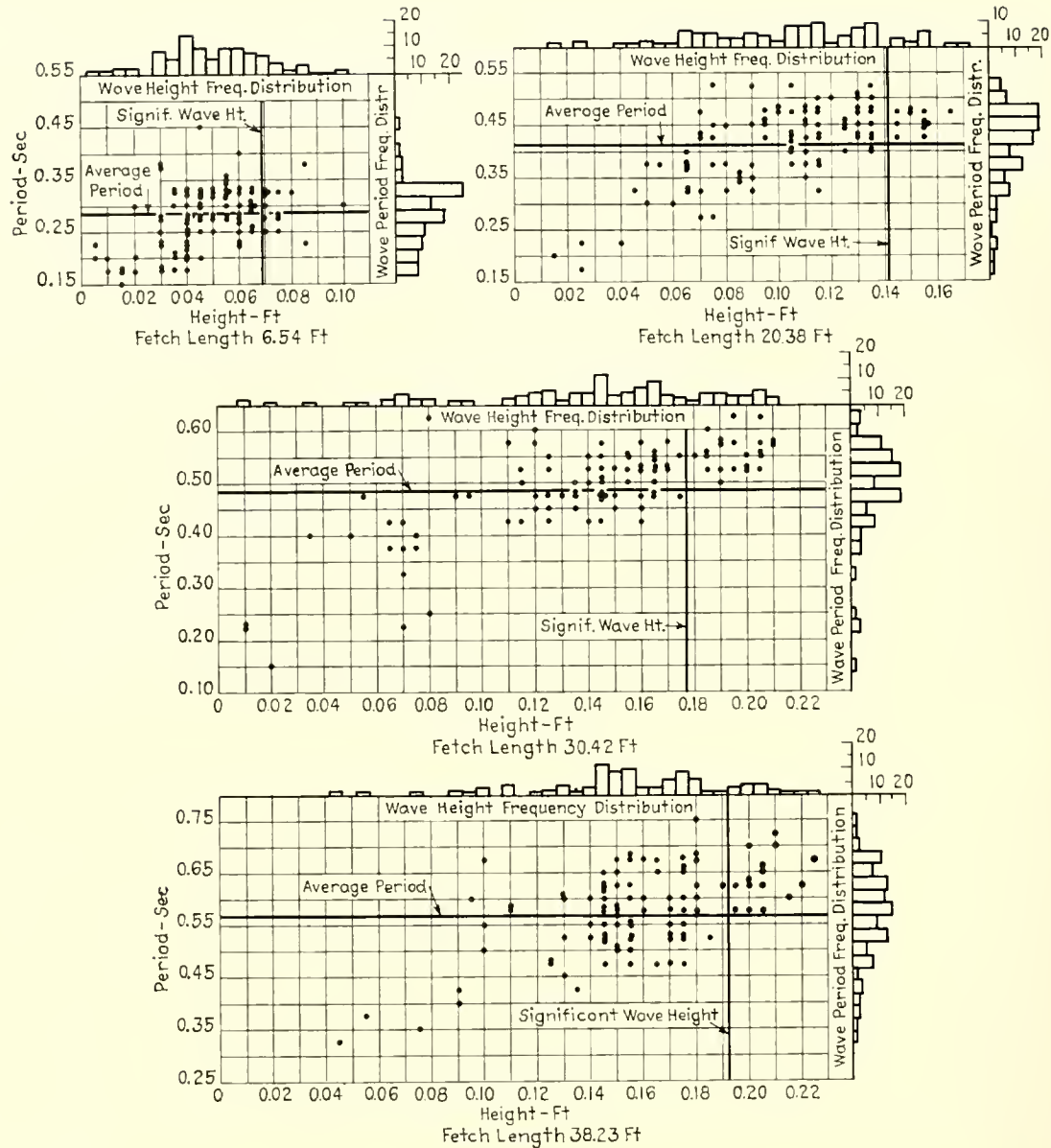


Fig. 12 Joint frequency distribution of wave period and wave height ($U = 42.7$ fps) (from Johnson and Rice, 1952)

and took for U the measured wind velocity at the height of 30 m (about 100 ft). In the published works on oceanography heights of 6 and 10 m (19.6 and 33 ft) are most often used. For measurements made on small waves with small fetches and light winds, Roll (1949) used U as measured at the height of 1 or 2 m. For small waves in a wind flume Francis (1951) measured U at $z = 10$ cm (0.33 ft), while Johnson and Rice (1952) took the mean air velocity in the wind flume. It should be noted that in a wind flume the air velocity first increases with height over the rough-water surface, and then decreases again in approaching the upper wall of the flume. The lack of an established similarity law and a definite convention is evident.

The relationship between wind velocity near a rough surface and the tangential drag τ per unit of surface area can be stated by quoting from Francis (1951):

"The velocity distribution within a boundary layer gives an indirect method of finding the shear stress on the boundary, and this has been used before to find the stress coefficient from field tests. The case of a turbulent flow over a solid rough surface is well known, and it has been shown that at a height z

$$u = 5.75 (\tau / \rho)^{1/2} \log_{10}(z/z_0) \quad (25)$$

where z_0 is a length expressing the effective roughness of the surface . . . By plotting u against $\log z$, a straight line results, of which the slope gives the value of

$5.75 (\tau/\rho)^{1/2}$, and the intercept at $u = 0$ gives $\log z_0$."

Under conditions of a fluid flow along a rough stationary surface, and in particular in the case of wind over land area analyzed by G. I. Taylor (1916), the surface does not absorb the energy. The loss of momentum in the air is accompanied by dissipation of the kinetic energy in eddies, turbulence, and finally in the form of heat. The organized kinetic energy of the potential air flow is in part disorganized, lost in the form of heat, and thus is no longer available. An entirely different situation exists in very mobile sea waves in which the energy is transmitted from air to water. To a large extent the air does work on the moving water surface by normal pressures, so that kinetic energy given up by the air reappears as the kinetic energy of the potential wave motion. Only a part of the kinetic energy of wind is dissipated in friction in the form of air and water turbulence. The theory of a boundary layer at a mobile or oscillating surface has not yet been developed, and so in practice it becomes necessary to assume that the turbulent boundary-layer relationships developed for fixed surfaces remain valid for the mobile water surface. The empirically derived coefficients, however, may not be the same in the two cases in view of the fundamental distinction of the two phenomena. Clearly realizing this distinction Neumann (1948, 1949a) speaks of the "effective tangential force coefficient," which is composed of contributions of both the dynamic, i.e., pressure, drag and the frictional drag, for which alone the turbulent-boundary-layer expressions are truly valid. This represents an assumption that the pressure drag of a moving, wavy surface affects the air-velocity distribution $u(z)$ in the same functional form as the frictional drag. Practical application of the method appears to confirm this assumption, but apparently no deeper investigation of this question and no crucial experiments were made. In connection with the foregoing Neumann (1948, 1949a) emphasized that the roughness parameter z_0 in equation (25) is a purely nominal quantity, characteristic of the sea surface but bearing no direct relationship to the apparent roughness of the sea. In fact, as will be shown later, the roughness parameter z_0 is often shown to decrease with increasing wind and apparent sea roughness.

In the computation of the drag of the earth's surface by G. I. Taylor (1916) the dimensions of the roughness (ground undulations, trees and so on) were small as compared to the height z used, and z was therefore obtained by measurements over the ground without ambiguity. In oceanography the usual measurements of the wind velocity are made at a relatively small height over large waves. A more specific definition of the height z is therefore needed. Roll (1948) expressed z as $z' + H/2$, where z' is the height above wave crests, and H the wave height. This definition is close to but not identical with measuring z over the undisturbed water surface. However, there was a kink in his plotted curve of $\log z$ versus u at a low value of z . Neumann (1949a) suggested that $z = z' + H$ be used; i.e., the lowest level of wave troughs be used as the reference level. In the

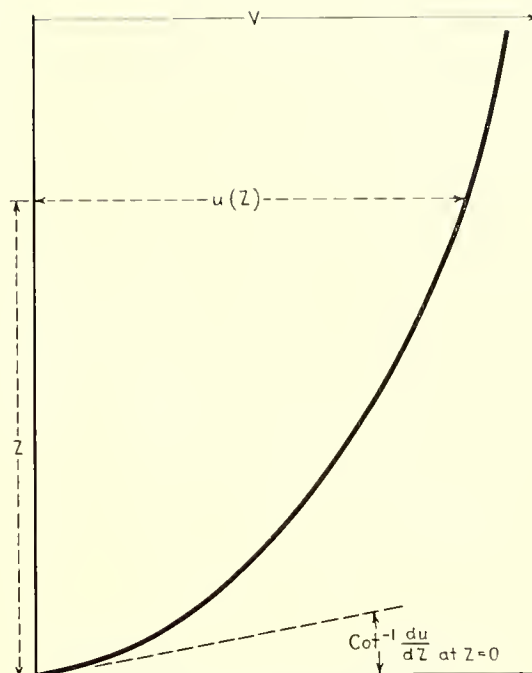


Fig. 13 Variation of mean air velocity versus height in vicinity of ground

plots made by him on this basis the kinks disappeared and the desired straight-line plot resulted. At a wind of 4 mps (about 13 fps) and wave height $H = 50$ cm (about 1.6 ft) shown by Roll's (1948) measurements, Neumann estimates $z_0 = 2$ cm (0.79 in.).⁷

Francis (1951) applied the method described in the quotation from his work given before to his measurements in a wind flume. At low z in proximity to the waves, however, the plot of u versus $\log z$ exhibited violent kinks, and only for a short range of the larger z heights measured was the plot linear. Although the measurements showed wide scatter, Francis obtained a mean value of 5.0 for the coefficient in equation (25), which is close to the theoretically expected 5.75. It was apparently impossible to evaluate z_0 (and therefore τ) from these plots.

3 Energy Balance in Waves and Energy Dissipation

The energy of a wave system grows with distance by the amount of the energy received from the wind less the amount dissipated by internal friction. An elementary analysis of this process will be given.

Consider a stretch of sea of unit width, traversed by imaginary control planes located at fetches F_1 and F_2 . The mean rate of energy gain E per square foot over the distance $F_2 - F_1$ is expressed as

$$dE/dx = \frac{E_2 - E_1}{F_2 - F_1} \quad (26)$$

⁷ For a recent discussion on properties of the boundary layer at the sea surface the reader is referred to Ellison (1956) (see p. 105).

where E_1 and E_2 are the energies per square foot per second carried over through the planes at F_1 and F_2 . From equation (60) of Appendix A

$$E_1 = (1/4)\rho g c_1 a_1^2 \quad (27)$$

in any consistent set of units, or

$$E_1 = 16 c_1 a_1^2 \quad (28)$$

in foot-pound units for sea water at $\rho g = 64$ pcf. Here c_1 is the wave celerity and a_1 the wave amplitude at F_1 . An identical expression with subscripts 2 will hold at F_2 .

The foregoing rate of change is equal to the difference between energy E_p , received from wind, and E_{ds} , dissipated in internal friction; i.e.,

$$dE/dx = E_p - E_{ds} \quad (29)$$

E_p is evaluated on the basis of normal pressures acting on water as

$$E_p = C_d \frac{\rho'}{2} (U - c)^2 c \quad (30)$$

where for "standard air" $\rho' = 0.00237$ pounds per cu ft.

The energy E_{ds} dissipated in internal friction is given for classical gravity waves by equation (66) of Appendix A in terms of the molecular coefficient of viscosity μ . G. I. Taylor (1915), in his study of atmospheric turbulence, introduced a coefficient of turbulent viscosity which is larger than μ . This coefficient will be designated by μ^* . Neumann (1949b) also shows that the effective turbulent coefficient of viscosity μ^* in wave motion is many times larger than μ . The turbulence responsible for this increase results partly from the energy transmitted from wind by skin friction and partly from the kinetic energy of wave motion dissipated in the process of the breaking of wave crests. Assuming for the present that the classical expression is valid with a new coefficient μ^* ,

$$E_{ds} = 2 \mu^* k^3 c^2 a^2 \quad (31)$$

where k is the wave number, $2\pi/\lambda$. Expression (31) is valid in any consistent set of units. In the foot-pound system, and taking $\mu = 2.557 \times 10^{-5}$ for sea water of 59 F (15 C), it becomes

$$E_{ds} = 0.065 n (a/\lambda)^2 \quad (32)$$

where n denotes the ratio μ^*/μ .

Since the wave height, $H = 2a$, and wave length λ are reported in all wave observations, the energies E_1 and E_2 can be computed readily. The energy received from wind, E_p , also can be computed, provided the drag coefficient C_d is known. C_d can be reasonably estimated from the data of the foregoing section. The only quantity completely unknown is the ratio n . It can be computed on the basis of equations (26) and (29) as

$$n = \frac{1}{0.065(a/\lambda)^2} \times \left[C_d \frac{\rho'}{2} (U - c)^2 c - (E_2 - E_1)/(F_2 - F_1) \right] \quad (33)$$

Data and computations for six cases found in the literature are shown in Table 4. This is but a small sample of data of varying reliability (for the present purpose) but nevertheless a few conclusions can be drawn:

a) There is little purpose in analyzing in this manner more of the data found in the current literature. Data must be obtained specifically with this type of analysis in mind for the results to be reliable.

b) The ratio n is not a constant but varies with (a/λ) ratio and with wave height. Expression (31) must therefore be modified by including a proper functional relationship for μ^* .

c) In the minute and mild waves (case 3) n is about 6. For essentially the same wave height but greater steepness and hence larger C_d in case 5, n increases to 41.

d) The n -values of 207 and 439 in cases 1 and 6 are apparently exaggerated by the excessive influence of the wave celerity c in the factor $(U - c)^2 c$ entering into E_p . In Table 4, the c -values are based on the reported mean or significant waves. It appears more probable that the energy-transfer calculations should be based on the smaller waves or ripples by which the significant waves are overlaid. This would call for smaller λ and c in the foregoing calculations.

e) If the same reasoning were applied to wave dissipation, the calculations also would have indicated greater energy dissipation, since smaller waves of lower c/U -ratio usually have higher a/λ -ratio.

f) Case 2 should be omitted. A comparison with case 1 shows too low a/λ for a similar c/U -ratio. Apparently, the waves performed by the wave generator were too long for the ambient wind conditions.

3.1 Energy Dissipation (by Bowden, 1950). The development indicated by paragraph b) of the foregoing section was attempted by Groen and Dorrestein (1950) who, on the basis of the previous work of Richardson (1926) and Weizsäcker (1948), assumed μ^* to be proportional to $\lambda^{1/3}$. Bowden (1950) showed that Weizsäcker's reasoning is not applicable to waves and, on the basis of dimensional reasoning, derived a new relationship. If μ^* depends on wave proportions, it should be a function of wave length, amplitude and period, so that

$$\frac{\mu^*}{\rho} = K \lambda^\alpha a^\beta T^\gamma \quad (34)$$

It follows that $\alpha + \beta = 2$ and $\gamma = -1$. Bowden took the simplest assumption that $\alpha = \beta = 1$ and wrote

$$\frac{\mu^*}{\rho} = K c a \quad (34a)$$

where K is a nondimensional coefficient. The rate of energy dissipation is then

$$E_{ds} = 2 \rho K k^3 c^3 a^3 \quad (35)$$

Bowden confirmed the foregoing results by a derivation based on von Karman's (1930a and b) similarity hypothesis for shearing flows.

The application of the foregoing equation to cases 1 to

Table 4 Estimated Energy Balance in Observed Waves

Case	$E_{2,1}$ ft	λ ft	c fps	a ft	$E_{2,1}$	$(E_2 - E_1)/$ $(F_2 - F_1)$	U fps	$(U - c)^2 c$ mean	C_d	E_p	a/λ mean	$0.065(a/\lambda)^2$ mean	$n =$ μ^*/μ
1	30.42 6.54	1.22 0.416	2.49 1.46	0.0935 0.0345	0.348 0.028	0.0131	42.7 12.7	3251	0.0258	0.100	0.080	0.00042	207
2	16.4 9.2	1.44 1.15	2.71 2.42	0.0935 0.0345	0.379 0.046	0.0462	39.3 39.3	3463	0.0214	0.088	0.047	0.00014	300
3	16.4 9.2	0.26 0.10	1.15 0.70	0.013 0.005	0.0031 0.0003	0.0004	16.4 16.4	219	0.0052	0.0014	0.050	0.00016	6.2
4	67.5 18.0	1.33 0.50	2.61 1.60	0.0637 0.0272	0.169 0.019	0.0030	19.5 18.6	603	0.0052	0.0038	0.051	0.00017	4.7
5	2.62 0.98	0.17 0.09	0.93 0.68	0.0112 0.0066	0.00187 0.00017	0.00085	25.5 25.0	482	0.0235	0.0136	0.069	0.00031	41.3
6	...	106	23.28	3.25	...	0	37	4370	0.0052	0.0272	0.031	0.000062	439
			(Steady Conditions)				(Est.)						

Legend:

Case 1. Johnson and Rice (1952)—Run 1a. C_d estimated as mean of first four lines of Table 3, assuming $C_r = 0.0028$ on basis of Table 1, Model 4.

Case 2. Francis (1951)—Severe Condition: Generator and Fan—12 mps. Data estimated from curves; $C_d + C_r = 0.0242$ from Table 2, C_r estimated as 0.0028.

Case 3. Francis (1951)—Gentle Condition: Fan only, 5 mps; $C_d + C_r$ listed in Table 2 would leave improbably small value for C_d . $C_d = 0.0052$ is assumed from Thijssse's (1952) experiments.

Case 4. Roll (1951)—Open Air Observations, Nos. 24 and 26. Conditions at two fetches and wind are means of several observations and do not represent a simultaneous set of conditions. C_d estimated as for Case 3. Roll's (1918a, Table 1) wind-profile measurements give the unreasonably low value of $C_d + C_r = 0.0025$.

Case 5. Roll (1951)—Observations 29 and 32. In shorter fetch and stronger wind than Case 4. In view of mean steepness of waves, C_d is assumed as the mean of Cases 1 and 2.

Case 6. Neumann (1950, page 41)—A typical sea condition in North-East trade winds quoted from Larisch-Wind Beaufort 5 to 6. C_d assumed from Thijssse's (1952) experiments.

6 of Table 4 yields values of K of 0.022, 0.022, 0.010, 0.0008, 0.073 and 0.000069, respectively. Bowden, on the basis of swell attenuation, gave the order of magnitude of K as 5×10^{-5} , which checks well with the figure computed for Neumann's case 6 in trade winds. The variability of the coefficient K indicates that the problem is not yet solved. It appears to the author that Bowden's dimensional reasoning was faulty in that T and λ are not independent, and λ and a are not dimensionally distinguishable. It is suggested that the derivation based on von Karman be investigated further.

3.2 Summary of the Energy-Balance Problem. Summarizing the foregoing, neither the problem of energy transmission from wind to waves nor the problem of energy dissipation in waves has been solved. The solution of the first hinges primarily on evaluating the celerity of the wavelets overlaying larger waves.⁸ The solution of the second requires theoretical and experimental determination of the function form of the turbulent viscosity μ^* . Flume and open-air wave observations have not been reported in a form useful for evaluating the wave-energy balance. New observations and flume tests are needed. These must be planned specifically with a suitable analytical procedure in mind. It cannot be too strongly emphasized that wind-flume and small-scale natural waves differ radically from natural ocean waves of significant size in the ratios c/U and a/λ . Small values of the former and large values of the latter predominate in small waves. Small-scale tests and observations, therefore, cannot be used as direct representation of open-sea conditions. The objective of the small-

scale observations should be, therefore, the development of rational relationships which then can be applied to any wave system.

The decrease of wave steepness a/λ with increase of wave length λ is one of the most reliable relationships observed in ocean waves. The quantitative relationship between these quantities was first noted by Sverdrup and Munk (1946, 1947) and later confirmed by Roll (1951) and Neumann (1950a and b, 1953a and b). Bowden appears to be the first to offer a rational explanation of this observed fact. His formula, equation (35), indicates a rapid increase of the energy-dissipation rate with wave height and length. The formula for the transfer of the energy from wind, on the other hand, depends on C_d which is a function of the wave form, but is independent of wave height. Thus the balance between the energy received and dissipated can only be achieved in higher waves at a lower a/λ value. While formula (35) appears to have exaggerated the influence of the wave size, the significance of this factor can scarcely be doubted.

The exposition of the energy balance in the growth of sea waves has been presented simply, in order to call attention to the basic elements of the problem. Both the transfer of the energy from the wind and the dissipation of the energy by turbulence and internal friction must be considered in defining the growth of waves. A procedure analogous to the foregoing, but much more elaborate, was employed by Neumann (1952a, b, c). It is reviewed in Appendix B. Although it is listed under "Practical Approach" because of the many empirical and intuitive steps involved, it is the most complete discussion of the fundamental principles involved in the energy balance in waves available to date. It served as a basis for the wave-forecasting method of Pierson, Neu-

⁸ A more exact formulation of this statement was given on page 9 (last paragraph) and it will be discussed further in Section 4.4.

mann, and James (H). On the other hand, the broadness of the subject is overlooked in the recent "advanced" work discussed in Section 4, and attention is concentrated entirely on the transfer of the energy from the wind.

4 Generation of Waves by Wind—Advanced Rational Approach

Under this heading several recent papers can be reviewed briefly. These are elaborate mathematical developments and the reader is referred to the readily obtainable original papers for the complete exposition of the subject. Only a general outline of the principles used will be given here, primarily in order to indicate the apparent shortcomings and desirable directions of further development.⁹

The advanced approach to the subject of wave formation by wind has taken two broad directions. Eckart (1953*a, b, c*) and Phillips (1957) took air-pressure fluctuations in a gusty wind as the primary cause of wave formation. The pressure fluctuations in this case are uncorrelated with the wave form. In another approach, Munk (1955) and Miles (1957), following in principle the elementary method of Jeffreys (Section 2), considered the air pressures as caused by the air flow about the wave profile. In this case the air-pressure variations are completely correlated with the waves. Two such different concepts can be entertained only in the early stages of development of a subject. It is probable that at a later stage of the theoretical development a concept of partial correlation will be introduced. In these early stages of development, also, the energy dissipation in waves has not been considered.

4.1 Eckart's Theory. The wind pressure is assumed to be everywhere normal to the undisturbed surface of water and caused by an ensemble of "gusts." A gust is defined as an area of high or low pressure, which moves with the mean wind speed, and has a radius $D/2$ and a duration or life T . At the end of the time period T a gust "blows itself out" leaving its wake to dissipate in the form of free gravity waves. First, the theory of this phenomenon is developed for a single gust. Next the effect of the ensemble of gusts is treated on the basis of the time average, as commonly used in the theory of turbulence. A very large number of gusts of uniform diameter and intensity is assumed to be distributed at random through space and time. This ensemble of gusts makes a storm.

Quoting from Eckart (1953*c*): "In the generating area the waves may be many meters high, and thus represent a large surface density of energy. This energy cannot be supposed to have been obtained from the air instantaneously and locally. Much of it will have been obtained from the air earlier and at a considerable distance from the point of observation (though still in the storm area). It will have been transported by the water essentially according to the laws of free wave motion.

This has long been recognized by the use of the concept of fetch . . ." The effect of the few gusts near to a point of interest on the sea surface is, therefore, negligible compared to the many distant gusts the effect of which has accumulated with fetch.

The waves caused by the ensemble of randomly distributed and fluctuating pressure areas represent the summation of many component waves of different wave lengths, heights, phases and directions of propagation. Such waves are described by a spectrum. As will be discussed in greater detail later, the resultant appearance of the sea is that of groups of waves separated by calmer regions, each group consisting of a few waves of varying heights. Quoting further from Eckart (1953*c*): "Since the surface disturbance has a random character, no unique value of wave number and frequency can be assigned to it; these dominant values correspond roughly to a maximum in the spectrum." From a consideration of the empirically observed number of 5 to 10 waves in such groups, Eckart concluded that in a wind of 20 mps (about 39 knots) for instance, the life T of the gust is 15 to 30 sec, and the typical gust radius $D/2$ is 40 m (130 ft).

Eckart's solution covers regions inside and outside of the storm area; the latter case is simpler and the solution is more precise. Outside the storm area the spectrum of wave directions is symmetrically disposed with respect to the radial direction from the storm center, i.e., the velocity of propagation of the dominant wave is in the radial direction. The existence of a spectrum of wave directions causes wave short-crestedness, and at the radial distance r of 10 storm diameters D , for instance, the average length of wave crests is 2.2 of wave length λ (between succeeding crests).

Inside the storm, the wave components have not yet separated, and there is no similarly dominant direction. Each point is traversed by waves travelling in many directions. The conditions are particularly confusing in the center of the storm area. The predominating direction of wave propagation becomes more clearly defined as a point under consideration moves from the center to the periphery of the storm area. Generally, the sector of wave directions inside the storm is not symmetric with respect to the wind direction, the asymmetry decreasing toward the edges of the storm.

Outside of the storm area the wave remains constant, since it is no longer influenced by atmospheric disturbances. Inside the storm area the formulas derived by Eckart imply that the spectrum is a function of position in the area, and in particular, that the effect of a given fetch depends on its position in the storm area.

While the theory developed by Eckart explains many of the observed characteristics of storm-generated waves, it fails to predict correctly the wave height. The air pressure needed to generate the observed waves according to the theory is shown to be possibly ten times greater than its probable value.

One of the possible reasons for this discrepancy is contained in the initial formulation of the problem by

⁹ Only in the realm of basic ideas. Mathematical techniques will not be discussed.

Eckart. The storm is modeled mathematically by the succession of *traveling pressure areas*, and no consideration is given to the effect of the *wind velocity*. The wind velocity U enters into Eckart's theory only in that the pressure areas are assumed to travel with the wind velocity. Neglect of the wind velocity itself is consistent here with the initial assumption of "infinitesimal" waves. Quoting again from Eckart: "The term infinitesimal in this connection, refers to the neglect of nonlinear terms in the hydrodynamic equations; it seems certain that this must be ultimately remedied if all phenomena connected with the interaction of wind and water are to be treated theoretically . . ." Particularly important in this definition are the infinitely small slopes of the water surface. Since the wind action on water appears to depend on the square or higher power of these slopes, the mechanism of the kinetic-energy transfer from wind to water is essentially absent in Eckart's case; only changes of static pressure of the air are considered. It should be clear from the preceding sections that the transfer of energy from wind depends specifically on the finite height of waves, and particularly on the existence of sharp slopes which are not considered in this case.

While the foregoing paragraph represents a plea for consideration of wave-correlated pressure areas, it should be noted that Eckart's conclusions (except for wave height) probably would not be greatly affected by such considerations. Since the waves themselves are randomly distributed, and in fact have the appearance of randomly distributed groups, it can be assumed that consideration of correlated pressures would have taken a form similar to the one used by Eckart. An apparent major difference would be the use of wave-group velocity instead of wind velocity for the propagation of pressure areas. Most of Eckart's conclusions, except the mean value of wave heights, can therefore be assumed to be valid for the actual sea surface, at least for the time being. Unfortunately, observational data on the angular dispersion of wave propagation and on the lengths of wave crests are very meager. In particular, data on the waves within the storm area appear to be almost completely absent. Such data as for instance Weinblum's (3-1936) stereophotographs on the *San Francisco* in a severe storm have not yet been analyzed in a form suitable for comparison with the theory discussed here. Furthermore they usually cover too small an area to be valid statistically.

The work of Eckart (1953a, b, c) can be considered as extremely important not only for its results, but for the method of attack as well. Further work based on this method, but considering the action of the wind on the irregular sea surface with large wave slopes, should be encouraged and sponsored.

4.2 Phillips' Theory. Apart from the mathematical methods, Phillips' theory differs from Eckart's by the adoption of a more general randomness. While Eckart postulated random distribution of gusts in space and time, he, by independent reasoning, specified the diam-

eters and durations of gusts. He also assumed that gusts move with the wind velocity U . Phillips made the statement of the problem more general by assuming that the dimensions and lifetime of gusts are also random. This included the smaller gusts moving near the sea surface in the air stream of reduced velocity $u(z) < U$. In stating the problem Phillips, therefore, postulated an (as yet) unknown velocity U_c .

A random distribution of fluctuating pressures is described by a spectrum;¹⁰ i.e., it is thought of as composed of a superposition of sinusoidally varying pressure fluctuations of different amplitudes, frequencies and phase relationships. The term "spectrum" or, more exactly, "spectral density" is applied to the mean amplitudes of fluctuations within a narrow frequency band. Waves excited by such pressure fluctuations also are described by a spectrum; i.e., by the summation of sinusoidal wave components of various amplitudes and frequencies. The end result of Phillips' solution is an expression defining the wave amplitudes in terms of the amplitudes of pressure fluctuations at all frequencies. This relationship is time dependent and the wave amplitude is shown to increase in proportion to the elapsed time. In the process of solution, the effective gust travel velocity U_c was defined.

Quoting Phillips, "It is found that waves develop most rapidly by means of a resonance mechanism which occurs when a component of the surface pressure distribution moves at the same speed as the free surface wave with the same wave number.

"The development of the waves is conveniently considered in two stages, in which the time elapsed [from the onset of a turbulent wind] is respectively less or greater than the time of development of the pressure fluctuations. An expression is given for the wave spectrum in the initial stage of development, and it is shown that the most prominent waves are ripples of wavelength $\lambda_{cr} = 1.7$ cm, corresponding to the minimum phase velocity $c = (4gT/\rho)^{1/4}$ ¹¹ and moving in directions $\cos^{-1}(c/U_c)$ to that of the mean wind, where U_c is the 'convection velocity' of the surface pressure fluctuations of length scale λ_{cr} or approximately the mean wind speed at a height λ_{cr} above the surface. Observations by Roll (1951) have shown the existence, under appropriate conditions, of waves qualitatively similar to those predicted by the theory.

"Most of the growth of gravity waves occurs in the second, or principal stage of development, which continues until the waves grow so high that nonlinear effects become important. An expression for wave spectrum is derived, from which the following result is obtained:

¹⁰ An outline of the notation and mathematics used in connection with random processes (particularly sea waves) will be found in Section 8. The reader is asked to accept the brief and incomplete statements on the subject in Sections 4 and 5, and thus a temporarily incomplete understanding of these sections, with the hope that he will return to them after perusing Section 8, "Mathematical Representation of the Sea Surface."

¹¹ T here is the surface tension.

$$\overline{\eta^2} = \frac{\overline{p^2} t}{2 (2 \rho^2 U_c g)^{1/2}} \quad (36)$$

where $\overline{\eta^2}$ is the mean square surface displacement, $\overline{p^2}$ the mean square turbulent pressure on water surface, t the elapsed time, U_c the convection speed of the surface pressure fluctuations, and ρ the water density. . . .

"We are now in a position to see rather more clearly the probable reason for the failure of Eckart's theory to predict the magnitude of the wave height generated by the wind. His less precise specification of the pressure distribution has 'smoothed off' the resonance peak of the response of the water surface, and it is the wave numbers near the peak that can contribute largely to the wave spectrum at large durations."

Application of equation (36) requires knowledge of the mean pressure fluctuation $\overline{p^2}$. Quantitative data on the turbulence in the boundary layer of the wind at the sea surface are meager and uncertain. However, Phillips used certain plausible data and evaluated $\overline{p^2}$ and $\overline{\eta^2}$ as functions of the elapsed time. He was thus able to demonstrate excellent agreement of wave-height growth versus time with the data of Sverdrup and Munk (see Section 5.1). The author believes, however, that this comparison is premature and has little meaning, since dissipation of the energy in waves has not been considered. It is evident that Phillips made a major contribution to the subject of wave generation by wind. He has ably treated, however, only one facet of the problem. This must be combined with other aspects (wave-correlated pressures, energy dissipation) before a comparison with observed waves can be meaningful. Phillips' results may be directly applicable to the initial formation of small ripples, at which time the energy dissipation depends on the molecular viscosity and is small, and the drag coefficient C_d and therefore the wave-correlated pressures are also small. The application of Groen and Dorrestein's (1950) and Bowden's (1950) results showing that energy dissipation grows with wave height and length may limit the indicated wave growth and eliminate the need of uncertain reference to nonlinearities.

In his 1958 work, Phillips defined by dimensional reasoning the theoretical shape of the high-frequency end of a wave spectrum. This definition was based on the observed occurrence of sharp-crested waves, the physical definition of a sharp crest by the vertical water acceleration $\ddot{\eta} = -g$, and the mathematical-statistical definition of a discontinuous function expressing the water surface elevation. Phillips found the spectrum¹² to be

$$E(\omega) = \alpha g^2 \omega^{-5} \quad (36a)$$

where α is a constant, g the acceleration of gravity and ω the circular frequency.

4.3 Statistics of the Sea Surface Derived from Sun Glitter (Cox and Munk 1954a, b). This work, describ-

ing the method and the results of observations at sea based on the statistical theory and outlining certain important relationships of this theory, serves as one of two basic constituent parts of the work of Munk (1955), to be discussed in the next section.

The following resume is abstracted from Cox and Munk (1954a): If the sea surface were absolutely calm, a single mirror-like reflection of the sun would be seen at the horizontal specular point. In the usual case there are thousands of "dancing" highlights. At each highlight there must be a water facet, possibly quite small, which is so inclined as to reflect an incoming ray from the sun towards the observer. The farther the facet is from the horizontal specular point, the larger must be its slope in order to reflect the sun's rays back to the observer. The distribution of the glitter pattern is therefore closely related to the distribution of surface slopes.

In order to exploit this relationship plans were laid in 1951 for co-ordination of aerial photographs of glitter from a B-17G plane with meteorological measurements from a 58-ft schooner, the *Reverie*. One of the objects of this investigation was a study of the effect of surface slicks. In the methods adopted oil was pumped on the water. . . . With 200 gal of this mixture, a coherent slick 2000 by 2000 ft could be laid in 25 min, provided the wind did not exceed 20 mph. Two pairs of aerial cameras, mounted in the plane, were wired for synchronous exposure. Each pair consisted of one vertical and one tilted camera with some overlap in their fields of view. One pair gave ordinary image photographs for the purpose of locating cloud shadows, slicks, and vessels; this pair also gave the position of the horizon and the plane's shadow (to correct for the roll, pitch, and yaw of the plane). The other pair of cameras, with lenses removed, provided photogrammetric photographs.

The method consists essentially of two phases. The first identifies, from geometric considerations, a point on the sea surface (as it appears on the photograph) with the particular slope required at this point for the reflection of sunlight into the camera. This is done by suitable grid overlays. Lines of constant α (radial) give the azimuth of ascent to the right of the sun; lines of constant β (closed or circumferential curves) give the tilt in degrees.

The second phase interprets the average brightness of the sea surface (darkening on the photometric negative) at various α - β intersections in terms of the frequency with which this particular slope occurs. On the density photographs the glitter pattern appears as a round blob with a bright core (on the positive print) and a gradually diminishing intensity to the outside. The density of the blob on the negative is then measured with a densitometer at points which correspond to the intersection of appropriate grid lines.

The results are expressed as the mean of squares of wave slopes in up-down wind direction σ_u^2 , and in cross wind direction σ_c^2 . The data on the observed waves and on measured mean squares of slopes are given in Table 5. This table represents an abstract of data from Cox and Munk (1954a), Table 1, with columns of λ/H and

¹² The reader is referred to Sections 6 and 8 for the discussion of wave spectra.

Table 5 Wave Data Obtained in Sun-Glitter Observations of Cox and Munk (1954)

Photograph designation	Wind V , ^a fps	Significant Waves ^b			Ratios		Mean square slope components		
		Period T , sec	Celerity, c , fps	Length, λ , ft	Height, H , ft	λ/H	c/V	Transverse σ_c^2	Wind direction σ_u^2
Clean Water Surface									
28 Aug. b	33.3	4	20.5	81.9	3.5	23.4	0.62	0.0211	0.0390
28 Aug. p	40.0	5	25.6	128	6	21.4	0.64	0.0294	0.0484
28 Aug. u	41.5	5	25.6	128	6	21.4	0.62	0.0287	0.0452
28 Aug. v	41.3	5	25.6	128	6	21.4	0.62	0.0276	0.0404
3 Sept. j	1.2	3	15.4	46.1	1.5	30.7	12.9	0.00337	0.00489
3 Sept. q	25.8	3	15.4	46.1	2	23.0	0.60	0.0224	0.0230
4 Sept. k	12.3	2	10.2	20.5	1	20.5	0.83	0.00694	0.00977
4 Sept. n	24.0	3	15.4	46.1	2	23.0	0.64	0.0136	0.0191
4 Sept. r	19.0	3	15.4	46.1	4	11.5	0.81	0.0134	0.0170
4 Sept. v	19.5	3	15.4	46.1	4	11.5	0.79	0.0136	0.0186
4 Sept. y	14.8	3	15.4	46.1	4	11.5	1.04	0.0172	0.0174
5 Sept. b	5.4	4	20.5	81.9	3	27.3	3.80	0.00534	0.00906
5 Sept. g	6.5	4	20.5	81.9	3	27.3	3.16	0.00609	0.00875
5 Sept. j	10.2	4	20.5	81.9	5	16.4	2.00	0.0102	0.0125
6 Sept. c	32.5	4	20.5	81.9	4	20.5	0.63	0.0252	0.0265
6 Sept. k	31.2	4	20.5	81.9	4	20.5	0.66	0.0254	0.0357
6 Sept. q	35.5	4	20.5	81.9	5	16.4	0.58	0.0254	0.0374
10 Sept. r	16.0	3	15.4	46.1	3	11.5	0.96	0.0137	0.0179
11 Sept. l	15.1	3	15.4	46.1	2	23.0	1.02	0.0136	0.0137
17 Sept. e	29.5	3	15.4	46.1	4	11.5	0.52	0.0209	0.0264
17 Sept. c, h, k, n, q	29.3	3	15.4	46.1	4	11.5	0.52	0.0230	0.0322
17 Sept. A	31.0	3	15.4	46.1	5	9.2	0.50	0.0224	0.0365
Water Covered by an Oil Film									
6 Sept. c	32.5	4	20.5	81.9	4	20.5	0.66	0.0111	0.0108
10 Sept. k	25.7	3	15.4	46.1	3	15.4	0.60	0.0102	0.0117
10 Sept. m	21.6	3	15.4	46.1	3	15.4	0.71	0.00860	0.0100
10 Sept. r	16.0	3	15.4	46.1	3	15.4	0.96	0.00967	0.00985
11 Sept. f	15.2	3	15.4	46.1	2	15.4	1.01	0.0107	0.0109
13 Sept. e	7.1	5	25.6	128	4	32	3.60	0.00391	0.00467
13 Sept. f	7.1	5	25.6	128	4	32	3.60	0.00724	0.00957
17 Sept. e	29.5	3	15.4	46.1	4	11.5	0.52	0.0106	0.0126

^a Wind was measured at the heights of 9 and 41 ft. The mean value, corresponding approximately to the often-used "anemometer height" of 24 ft is listed here.

^b The wave periods and heights were recorded. The listed values of wave length λ and celerity c are obtained from Table 3 of Appendix A for the corresponding periods.

c/V added. The σ^2 are shown plotted against wind speed in Fig. 14 and the probability distribution of σ^2 is shown in Fig. 15. This latter will be discussed later in connection with the statistics of the irregular sea.

The major conclusions can be given by a partial quotation from the summary of Cox and Munk (1954):

a) As a first approximation the slope distribution is found to be Gaussian; this can be accounted for by an arbitrarily wide continuous spectrum of ocean waves, but not by a spectrum consisting of a few discrete frequencies.

b) The ratio of the up/down wind to crosswind components in the mean square slope is less than two; this indicates the directional "beam width" in excess of 130 deg for the relatively short waves that constitute the slope spectrum.

c) The mean square slope, regardless of direction, increases linearly with wind speed and reaches a value of $(\tan 16^\circ)^2$ for a wind speed of 14 mps (about 45 fps or 27 knots); this empirical relation follows in form and to an order of magnitude from a spectrum proposed by Neumann on the basis of wave-amplitude observations. A

spectrum proposed by Darbyshire cannot be reconciled with our observations.

d) Oil slicks laid by the vessel over an area of $1/4$ sq mile reduce the mean square slope by a factor of 2 or 3.

Several conclusions of lesser importance for the present monograph pertain to statistical properties of records and are omitted in the foregoing quotations. The discussion of statistics and spectra will be deferred until later chapters on irregular seas, and only the features bearing on the next section, on Munk (1955), will be discussed here.

This work is extremely valuable in introducing an ingenious technique based on statistics, in providing very valuable data on the sea surface, and in discussing statistical relationships for the sea surface. The conclusions appear, however, to be too broadly stated in some respects, without regard to the limitations of the relatively small scope of observations. An examination of the wave data in Table 5, particularly of the columns of λ/H and c/V shows that the observed significant waves are mostly either too small or too large in relation to the observed wind. The former case indicates that either

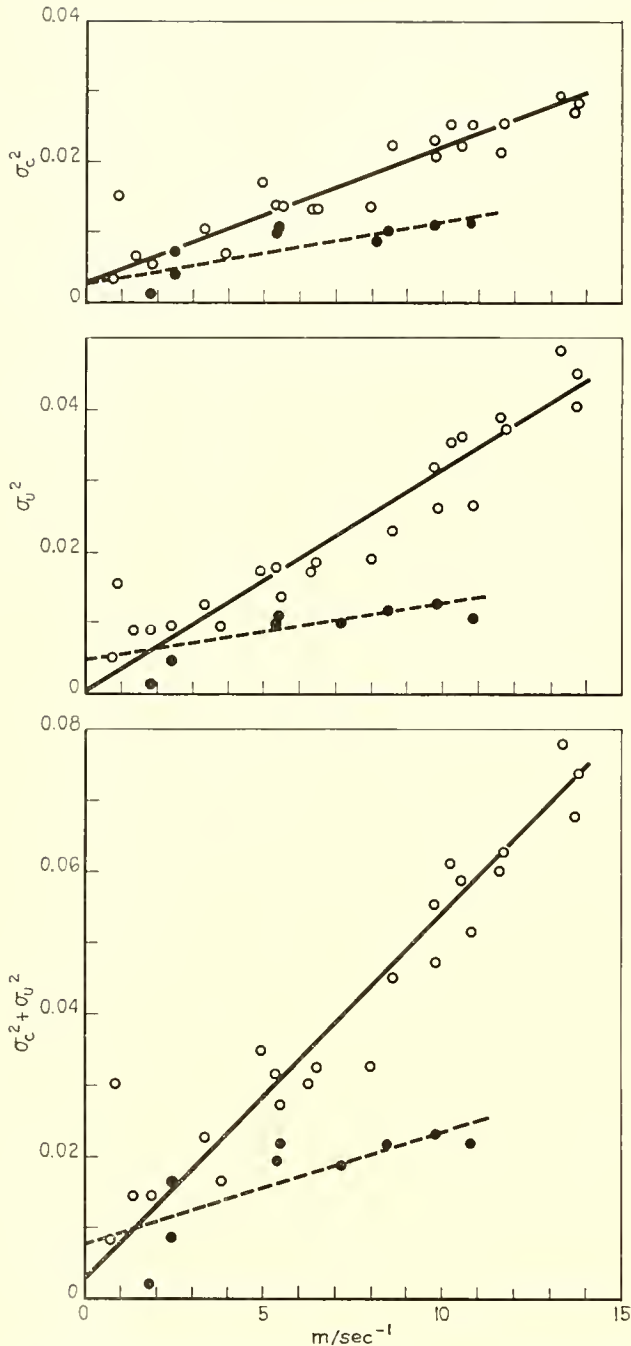


Fig. 14 Mean-square, wave-slope components and their sum as functions of wind speed. Open circles and solid lines for clean sea surface, solid dots and dotted lines for detergent covered (slick) surface (from Cox and Munk, 1954)

and the wind sea, but such a distinction is missing in the work under consideration.¹³

A ratio of squared slopes σ_u^2/σ_v^2 of about 2.5, indicating a directional spread of waves of about 130 deg, applies essentially to the small waves by which the larger observed waves are overlaid. Spreading of an oil film eliminated these small waves, leaving the larger waves unaffected. It is surprising to find that the ratio σ_u^2/σ_v^2 in this case is reduced to nearly unity, indicating an increased degree of short-crestedness. A possible explanation is that several swells of different directions (independent of the wind) were present, while the small and steep waves were caused by the local wind.

Examination of Table 5 and of Fig. 14 shows clearly that steep wave slopes are connected with the small waves by which the larger observed waves are overlaid. The recorded slopes are drastically reduced when these small waves are eliminated by the oil film. The mean square values of slopes σ^2 are seen to have little relationship to the observed wave dimensions, since these small waves are neglected in the definition of the "significant wave" as the mean of the $1/3$ highest waves. On the other hand σ^2 is seen to depend directly on the wind strength.

The conclusion that σ^2 is proportional to the wind velocity, as shown by Fig. 14 and as stated by Cox and Munk (1954a) may, however, be misleading. This relationship is shown to exist within the scope of observations, but the wave slopes cannot increase indefinitely, and the statistical observations should not be extrapolated without regard to the physical properties of waves.

4.4 Horizontal Drag Force Exerted by Wind—W. H. Munk's Hypothesis. The objective of this work is stated in the following quotation from Munk (1955a): "The problem of wind stress on water plays an essential part in studies of ocean circulation and storm tides, and of the momentum balance of atmospheric circulation. The present work is an attempt to connect results from recent experimental determinations of wind stress with the results from measurements of wave statistics . . ."

The starting point is the expression by Jeffreys (1925) for the pressure exerted by wind of velocity U on an element of the wave surface

$$p = s\rho'(U - c)^2 \partial\eta/\partial x \quad (37)$$

where s is a coefficient called by Jeffreys "sheltering coefficient" and assumed to be constant. The horizontal component of this pressure (i.e., drag force or wind stress) is

$$\tau = s\rho' \langle (U - c)^2 (\partial\eta/\partial x)^2 \rangle \quad (38)$$

where the symbol $\langle \rangle$ indicates that the mean value is taken.

The foregoing formulas were written for a simple harmonic wave, the celerity c of which is known. When the

¹³ A more complete description of the environmental conditions of these observations was published by Darbyshire (1956a).

the fetch or the wind duration were not large enough to give a fully developed sea. The latter case indicates that the observed significant waves were to a large extent due to the presence of a swell and not due to the local wind. In any study pertaining to waves it is very important to make a clear distinction between the swell

and must be of the type permitting mass analysis with the minimum of labor. The variability of sea conditions indicates that simplicity of data collecting and analyzing far outweighs the desire for extreme accuracy. In particular, the wave-energy distribution can be measured in only a few discrete directions. Gelei, Casalé, and Vassal expressed the mean directional spectrum by only two bands, Section 8.72.

29 Pending the collection of a sufficient amount of new wave measurements, the **Material Used for Spectrum Formulation in the Past Should be Re-examined**. In particular the following two projects can be suggested:

(a) The shape of Neumann's, Section 6.2, spectrum was obtained by intuitive analysis of a few wave records (see legend in Fig. 36). A formal spectral analysis of these records is recommended. In particular, this would help to clarify the evaluation of the constant C in Neumann's spectrum formulation, Section 6.23.

(b) Darbyshire's (1955), Section 6.11, wave data represent the only large collection of open-ocean instrumental wave measurements available to date. Re-examination of this material is recommended with particular regard to (i) verifying wind velocity over relevant fetches by re-examining meteorological conditions; (ii) including the necessary (theoretical) corrections of Tucker's gage indications, particularly for high-frequency waves; (iii) making independent spectral analyses, and (iv) experimenting with replacement of Darbyshire's empirical formula by alternate, possibly more sophisticated, formulations. In particular it is desired to bring out more clearly the effects of fetch length and of wind duration. Also, it is desirable to establish the spectrum form for a sea in the development stage, following the example of Gelei, Casalé, and Vassal (Sections 6.3 and 6.4; Fig. 45).

30 The collection and analysis of open-ocean data suggested in project 22) probably will take considerable time. Meantime appreciable progress can be made by **Spectral Analysis of Observations in Restricted Water Areas** of various sizes and at various wind velocities, project 14). A particular objective of this analysis would be to find out if the constant defining the spectrum area is truly a constant or whether it depends on other factors, particularly on "wave age" (i.e., the ratio of predominating wave celerity to wind velocity), verifying the conjecture made by the author in the fourth paragraph of Section 6.23.

31 **Development of Descriptive Wave Spectra**, Section 6.6 The development of a compact description of a sea surface is recommended. This should be suitable for ship-motion analysis or prediction. It appears that a three-parameter definition indicative of the wave height, wave period, and sea irregularity can be useful. Such a definition was given by Voznessensky and Firsoff's spectrum, Section 6.6. Additional work connected with it may consist of

(a) Evaluation of the three parameters for certain spectra of measured waves at sea, such as, for instance, Walden and Farmer's, Section 6.53(b). Attention

should be called to the fact that the spectrum in this case describes an observed sea which is usually generated by many separate causes. This spectrum therefore is basically different from the spectra based on wind velocity and a specified simple fetch and duration. In particular, the descriptive spectrum must have flexibility in specifying the dominant wave period.

(b) Preparation of a photographic album of various spectrally analyzed sea conditions for guidance in visual sea observations.⁴³ The photographs can be labeled and classified by the three Voznessensky and Firsoff parameters.

(c) Establishing relationships among the actual spectrum of waves, the Voznessensky and Firsoff three-parameter spectrum, simple measurements of wave heights and periods on a wave record, and the visually observed significant wave height and period. This can be accomplished by theoretical considerations based on mathematical statistics, in conjunction with the spectral evaluation of wave records (in subprojects *a* and *b*). The primary objective of this project is to enlarge the collection of wave data reducible to spectral presentation, which is needed in the prediction of ship motions.

(d) A special study to bring out the physical significance of the sea irregularity parameter α in Voznessensky and Firsoff's formulation. There is some evidence that in a "young sea" (large U/c) both the scalar spectrum and the directional energy distribution are broader than in the case of a fully developed sea. This observation apparently permits the grouping of period irregularity and of short-crestedness under one parameter. The foregoing is a conjecture, however, and the subject must be investigated.

32 **Synoptic Wave Data.** Collection of ocean-wave data on a synoptic basis is needed and is sure to come with time. A compact (and at the same time significant) method of reporting the wave data must be developed. To describe a complete spectrum a large number of ordinates must be reported. Any number of discrete ordinates will create uncertainty because the spectral form resulting from a computational procedure is usually irregular. Furthermore classifying a large number of reported spectra presents a problem. It does not appear to the author that it will be practical to report the complete spectra. The spectrum forms most often observed may be reported with sufficient accuracy by the three Voznessensky and Firsoff parameters which also provide a simple means of wave classification. The less frequent two-maxima spectra can be approximated by superposition of two simple spectra; i.e., by a total of six numbers. This demonstrates a possible approach. A broad study of the problem is recommended.

33 **Spectra of Very High-Frequency Wave Components.** The spectra discussed in Section 6 are macroscopic descriptions of sea waves. They give information on waves sufficiently long to be significant for ship

⁴³ This idea of a photographic album was advanced at the August 6 and 7, 1958 meeting of the Seakeeping Characteristics Panel of SNAME.

motions and for swell forecasting. The resolution power of the instrumentation used in their measurement and of their spectral analysis is not sufficient to describe small wavelets by which the surface of the larger waves is covered. These spectra, therefore, appear to be of doubtful value in problems of energy transfer from wind to waves. This transfer appears to be primarily dependent on the small sharp-crested waves. Projects on the measurement and analysis of these small waves are therefore recommended as a prerequisite to understanding the wave growth under wind action.

How small is "small" in this connection is not known now, and it is probable that it is defined not by absolute value but in relation to the total spectrum energy. It appears to the author, however, that waves in the gravity range are involved here, and one should not arbitrarily identify "small" waves with capillary waves.

A study of properties of the high-frequency end of the wave spectrum by Phillips (1958) can be mentioned as an example. The prospective investigator should be warned that the facet velocity of small waves depends not on their properties alone, but on the whole wave spectrum. Longuet-Higgins' (1956, 1957) papers can be used for defining the facet velocity in any spectrum. It also should be remembered that the very high-frequency ends of all of the spectra listed in Section 6 represent an extrapolation of the empirical data, and are therefore not reliable. Pending actual measurement of very high-frequency components of moderate and high seas, it is suggested that hypothetical spectra be used in theoretical studies. These would be composed of the spectra listed in Section 6 with the high-frequency ends replaced by Phillips' (1958) formulation. As sub-projects under the foregoing, the following can be listed:

(a) Measurements of the very high-frequency end of the spectra by auxiliary instrumentation at the time the usual wave measurements are made. In particular, the small wavelets can be measured with respect to a fairly large buoy riding on larger sea waves which in turn are recorded by accelerometers.

(b) Verification of Phillips' (1958) formulation by application to Cox and Munk's, Section 4.3, and Schooley's (1958) wave slope spectra.

(c) Carrying out preliminary work on the wind-energy transfer to waves using spectra as indicated in project (14-b) but with a hypothetical spectrum possessing Phillips' high-frequency extension.

34 Transformation of Small Waves into Large Ones. In the development of waves under the action of the wind there appears to exist a perpetual change from small waves to large ones. Sverdrup and Munk and Neumann (prespectrum) have shown that waves must grow in length in absorbing the wind energy since they cannot grow indefinitely in height. The detail mechanism by which the small waves are transformed into large ones is, however, not known. Efforts to formulate and to demonstrate a suitable theory are recommended.

The energy transfer from wind appears to depend on

the action of small waves. In comparison with these, the long waves of moderate sharpness and approximately trochoidal form have very small ability to absorb wind energy. The theory of large-wave growth must apparently depend on understanding the processes by which the wind energy absorbed by small sharp-crested waves is transformed into the energy finally appearing in large waves.

The reverse problem also has been observed in towing tanks. Sometimes an apparently regular wave train, after running through a certain distance, is transformed into an irregular wave pattern containing components of much higher frequency than the original waves.

35 Manuals of Applied Mathematical Statistics. The introduction of spectral descriptions of waves and ship motions brings about the need for knowledge of mathematical statistics. It can hardly be expected that practical oceanographers, ships' officers, and naval architects will have the time and preparation for a profound study of this subject. It is recommended, therefore, that short and simple texts on the relevant aspects of mathematical statistics be prepared. The text should have direct practical use as an objective, and should avoid theoretical discussions beyond those immediately needed for understanding the practical procedures. Those engaged in research requiring a deeper theoretical insight would be directed to the many existing textbooks and articles on a higher mathematical level. The notation and expressions familiar to oceanographers and naval architects should be used as far as possible, and the unfamiliar terminology of the specialized statistical texts should be avoided. The text should preferably be arranged in a graded form, starting with the simplest possible use of mathematical statistics in oceanographers' and naval architects' problems, and progressing to the more elaborate ones.

36 Provision of Recording and Analyzing Facilities. The practical application of the spectral concepts of the sea surface requires widespread use of suitable recording and analyzing equipment. Heretofore, the work in this field has been carried out only on a pilot-research basis, making use of the few available computing centers and often resorting to tedious manual measurements of various records. Significant progress in practical utilization of the modern irregular-sea concepts depends to a large extent on widespread availability of suitable recording and analyzing equipment. The need for ship-borne equipment has already been mentioned under project (26). Suitable equipment also must be made available to experimenters in wind flumes, towing tanks, and oceanographic institutions. It is often impractical to develop the necessary instrumentation at each individual establishment because of the lack of specialized knowledge and because of the cost involved. There is also the danger that heterogeneity of the methods and of the forms of reported results will hinder progress. It is, therefore, recommended that steps to develop and make available suitable instrumentation be taken by the proper professional organizations singly or jointly. If

suitable equipment specifications were developed, avoiding unnecessary complications and costs, private industry may be able to produce the equipment in quantity. Simplicity, reasonable universality, and low cost are essential, since much of the valuable research is expected to come from many small laboratories. In general, two broad types of equipment are visualized:

(a) Magnetic-tape recorders and analyzers of the (moderately broad) filtering type.

(b) Digitizers which translate the continuous electric signals from the sensing elements into the punched-card or tape records suitable for direct use in universal computing machines. This equipment is particularly appropriate for small laboratories connected with universities or other institutions possessing high-speed digital computing machines.

The author believes that the most rapid progress in research in oceanography and naval architecture will be made if the analyzed test data could be available to a researcher while the physical observations are still clear in his mind. The electronic filtering technique listed under *a*) and recommended for project 26) gives promise of such a rapid analysis. By using transistor techniques it also gives promise of a compact and rugged equipment suitable for use on location in natural-wave observations.

The author realizes that recorders, digitizers, and analyzers have been developed by various laboratories and that many electronic components are available in the open market. Nevertheless, no complete, compact and workable instrument package appears to exist, and the cost and the needed specialized knowledge severely limit the activity in this field.

37 Clarification of Confidence Limits. The confidence limits of spectral analysis are defined with respect to certain, rather narrow, filters of the digital analysis or electronic devices. When these limits are given in the literature, as for instance in Fig. 71, it is often difficult to find the frequency band widths to which they apply. Furthermore, these particular frequency band widths may or may not be relevant to the problem at hand. No distinction has been made between confidence in an analysis of a particular wave record and confidence in this particular record considered as a sample of the random sea conditions. Finally, there appears to be some confusion in the literature between expressing the confidence in terms of wave-record-measurement subdivision and in terms of wave lengths available in a sample. Further research to clarify the situation is recommended.

It is emphasized that in practical use the confidence limits of the spectrum must be closely connected with the objective for which the spectrum is to be used. For instance, confidence limits may be desired in evaluating the significant wave height, i.e., the zero moment of the spectrum, or the mean wave slope (the second moment), or the mean wave period. A tabulation of such confidence limits appears to the author to be more valuable in practical problems than drawing the usual statistical confidence-limit curves. The problem of clarifying the

meaning of statistical confidence limits in application to practical problems requires the joint work of mathematical statisticians and oceanographers or naval architects.

38 Instrumentation for the Measurement of Directional Wave Spectra. The need for measurement of wave directional spectra on a quantity, i.e., statistical, basis has been indicated under project 28) and has been mentioned several times before. Development of the necessary instrumentation can be listed, however, as an independent project. To date, Barber's methods appear to be the only ones suitable for mass collection of data. Barber proposed several methods, but only one of these, the correlation one, was outlined in some detail in Section 8.72 where references also were given to all of Barber's papers. A certain rather obvious development of Barber's correlation method is needed for the collection of the data suggested in project 28):

(a) The directional spectrum should be obtained for several wave frequencies.

(b) The variability of sea conditions, demonstrated by Tucker, Section 8.44, requires that measurements for all frequencies be obtained simultaneously. Also it would be desirable to obtain simultaneously the records for several pairs of gages needed for correlation analysis, instead of the consecutive measurements used by Barber.

It appears to the author that available electronic techniques will permit the following scheme: (a) The usual wave-height recording can be made simultaneously for several pairs of gages on the same magnetic tape thus permitting a cross-spectral analysis to be made later; (b) the multiple record should be passed alternately through several frequency filters, yielding several single-frequency multiple-gage records; (c) each single-frequency multiple-gage record should be subjected to the correlation analysis described by Barber, except that electronic methods of analysis would be used instead of Barber's pendulum; (d) the randomly distributed two-dimensional correlation function, equation (149), would be projected on the oscilloscope screen and photographed; (e) the photograph for each frequency would be interpreted in the same way as Barber's photograph, Fig. 87.

It should be emphasized that sea-surface variability makes it unnecessary to describe in excessive detail a directional spectrum from a single observational run. The spectrum is a statistical concept and the typical spectrum is to be obtained as the mean of many individual records. This being the case, an excessive number of gage pairs and of calculated wave directions should be avoided. It appears to the author that four pairs of gages and four wave directions will be sufficient.

The instrumentation outlined in the foregoing may be adapted to the measurements made with conventional wave-height gages as well as to the wave-slope and wave-acceleration measurements by floating buoys. In the latter case the use of telemetering equipment will be required.

39 Directional Wave Spectrum Measurement From Ships at Sea. Apparently no method of measuring the directional wave spectrum on ships has been proposed to date. Nevertheless, effort should be applied to this problem.

40 Energy Transport in Irregular Waves (Section 8.8). The static concept of the wave-energy spectra may not be adequate in problems of the energy transfer from wind to waves. Thought must be given to the mathematical and physical consequences of the energy transport by irregular waves. Defined with respect to harmonic-wave components by the classical theory, the energy-transport concept should be generalized by statistical theory. The work suggested by this project can be considered as a further development of Longuet-Higgins' (1956, 1957) work with particular emphasis on flow (or transport) of energy in various directions.

41 Waves of Extreme Steepness and Their Properties. In the analysis of dangerous ship stresses it is important to know the maximum steepness of ocean waves of various lengths. A maximum height-to-length ratio of 0.14 and a minimum included angle of 120 deg at the crest are indicated by classical theory (Section 3.2 of Appendix A) for simple gravity waves. A minimum included angle of 90 deg is indicated (Taylor, 1953) for standing waves.

(a) Theoretical research is needed to establish the maximum steepness and the mean wave height for short-crested irregular waves. Conceivably, the interaction of various wave trains may bring about the reduction of the 120 deg angle. This angle is reduced to 90 deg for standing waves which are represented mathematically by a summation of two simple wave trains.

(b) Ship-stress analysis requires not only knowledge of the wave steepness as a function of wave length but also of the pressure distribution in waves. The methods by which the limiting crest angles were determined in simple gravity waves involved only local conditions and not the general flow description. A project in evaluation of pressure distributions in waves of limiting steepness is therefore recommended both for long-crested and for short-crested irregular waves. While the problem may prove to be prohibitively difficult for sharp-crested waves, the computations for Stokes' waves (Section 3.1 of Appendix A) are simple and will yield valuable data.

(c) The spectral sea description is based on the linear superposition of simple wave trains and, strictly speaking, is valid only for very low waves. Development of a nonlinear statistical wave description is needed to represent the waves approaching limiting steepness. This project consists of: (i) the basic development of nonlinear methods and (ii) their application to typical sea spectra. In defining the latter, Bowden's, Section 3.1 limitation of the wave steepness by the balance of the energy received from wind and dissipated in waves and Phillips' (1958) definition of sharp wave crests by the condition that $\ddot{\eta} = -g$ may be useful.

(d) Expressions for the wave slopes are available in the statistical work of Pierson and Longuet-Higgins. These,

however, are based on the linear theory. A study of storm-wave records (for instance, Darbyshire's, 1955) is recommended in order to verify empirically the degree of agreement between large wave slopes as observed and as derived from linear spectra. Wave steepness appears to be connected with wave age, c/U , so that small-scale data, as in Cox and Munk's, Section 4.3 sun-glitter measurements, are not applicable to the present project. It must be based on full-scale storm conditions.

42 Shape of Wind-Driven Waves. The shape of wind-driven storm waves may be of significance in evaluating the bending moments acting on ships in waves. The increased steepness of leeward slopes can be expected to cause appreciable increase in the bending moments. Three subprojects are indicated here:

(a) Efforts to formulate and solve the problem theoretically.

(b) Empirical evaluation of the increase of the observed leeward slopes of storm waves over the slopes predicted from linear spectra.

(c) Empirical modification of the descriptive spectrum formulation (such as Voznessensky and Firsoff's, Section 6.6) to generate an unsymmetric wave form.

43 Restricted-Water Waves. Increased steepness of storm waves progressing into restricted waters (reduced depth, channel constriction, head current) may cause increase of ship stresses and is, in fact, suspected to be the cause of certain ship failures. Theoretical and empirical studies of the properties of these waves are desired. It is necessary to know the pressure-distribution pattern in these waves as well as their forms. The increase of sea severity near steep shores (for instance in the Bay of Biscay) is well known to mariners. It can generally be attributed to the standing-wave system caused by wave reflections from shore, but a more thorough quantitative investigation of the wave properties is needed.

44 Natural and Ship-Wave Interaction. The interaction of ship-made waves with natural wind waves can often lead to weird wave forms, excessive wave steepness and dangerous surf-like breakers. Two particular cases of interference-caused waves can be cited:

a) Interference of the following sea with the transverse stern wave of a ship. This may lead to the breaking of a large wave over a ship's stern (Möckel, 3-1953).

b) Interference of a ship's bow waves with oblique or beam seas. The interference breakers can often be seen over a large distance in the directions of the oblique bow and stern ship waves. This interference often increases ship wetness and may be dangerous for a ship's superstructure. A ship in a formation may be endangered by the combined interference of its own and other ships' waves with ocean waves.

Theoretical and experimental work on the properties of interference waves is recommended. This study can be expected to lead to the development of operational rules for increase of safety of fast (naval) ships operating in formation in rough weather.

Nomenclature for Chapter 1

- a = harmonic wave amplitude, half-height of trochoidal and Stokes' waves
 a_n, b_n, c_n = coefficients of the n th term of the Fourier series expansion of a function
 A_n, B_n, C_n = coefficients of the air-pressure-function expansion
 A, B, C = coefficients
 c = wave celerity (also known as phase velocity)
 C = a coefficient
 C_d = dynamic (i.e., pressure) drag coefficient
 C_p = pressure coefficient
 C_r = frictional-drag coefficient
 $C_d^* = C_d + C_r$ = total drag coefficient
 D = gust diameter (Eckart, Section 4.1)
 E = energy
 E = wave-energy content per unit of sea surface per second. Energy indicated by the spectrum area (true energy divided by ρg)
 $E(T), E(f), E(\omega)$ = spectral energy (divided by ρg) density in terms of wave period, frequency and circular frequency, respectively
 $E(u, v)$ = directional spectrum energy density in terms of wave number projections on x and y -axes
 $E(\omega, \theta)$ = directional spectrum energy density in terms of circular frequency and wave-propagation direction
 E_p = wave energy per unit sea area per second transmitted from wind to waves by normal air pressures
 E_{ds} = wave energy per unit area per second dissipated by waves
 $E_{1,2}$ = wave energies carried through reference planes at fetches F_1 and F_2
 f = frequency $1/T = \omega/2\pi$
 f = number of degrees of freedom in spectral analysis (equation 126)
 f_c = cut-off (Nyquist) frequency
 F = fetch
 g = acceleration of gravity
 h = water depth
 h = number of integrations of an autocovariance function in computing spectral density
 H = wave height
 \bar{H} = apparent wave height
 $\bar{\bar{H}}$ = mean apparent wave height
 $H_{1/3}, H_{1/10}$, etc. = mean height of $1/3, 1/10$, etc., of highest waves
 H = equivalent wave height (Darbyshire, Section 6.1)
 $i = \sqrt{-1}$
 k = wave number $= 2\pi/\lambda = \omega^2/g$
 L_h = "raw" value of spectral density
 m = maximum number of time lags in autocovariance analysis
 m_n = moments of scalar spectrum defined by equation (128a)
 m_{pq} = moments of the directional spectrum defined by equation (142)
 n = order of harmonic components in a Fourier Series
 n = number of intervals Δt in a sample of duration T
 n = generally an index or a subscript of a meaning to be specified
 n = ratio of turbulent and molecular viscosities, μ^*/μ
 N_0 = number of zero up crosses of a wave record
 N_1 = number of maxima of a wave record
 p = pressure
 p = number of lags, $\tau/\Delta t$
 p = index of the wave number u in directional-spectrum analysis
 q = index of the wave number v in directional-spectrum analysis
 q = successive numbering of ordinates of record of a random function
 r = proportion of negative maxima in a wave record
 $R(\tau)$ = autocovariance function
 $R(\tau)/R(0)$ = autocorrelation function
 R, θ = polar co-ordinates
 rms = root-mean-square
 s = Jeffreys' sheltering coefficient
 s = Miles' (Section 4.5) mass parameter
 S = setup of water surface caused by wind drag
 S_1 = setup of water surface smoothed by a detergent
 S_2 = setup of water surface caused by wave resistance
 t = time
 T = period of harmonic waves
 T = duration of a wave record, sec
 T = duration or life of a gust (Eckart, Section 4.1)
 T = surface tension
 \bar{T} = mean apparent period of irregular waves defined by number of zero up-crosses per second
 $\bar{\bar{T}}$ = mean apparent period of irregular waves defined as time intervals between dominant wave crests
 u = horizontal component of orbital velocity
 u = projection of the wave number k on x -axis
 $u(z)$ = wind velocity at the height z
 U = air velocity affected by the proximity of a body. Air velocity at anemometer height
 U = energy contained in a band of frequencies $\omega_2 - \omega_1$
 ΔU_T = spectral wave energy (in energy units) contained in the band ΔT of wave periods (Neumann, Section 6.2)
 U_c = gust travel velocity (Phillips, Section 4.2)
 v = vertical component of the orbital velocity
 v = projection of the wave number k on y axis
 V = velocity of undisturbed air. Gradient wind velocity
 V_c = formula velocity (Section 2.5)
 x, y = Cartesian co-ordinates
 $y(t)$ = an ordinate of a stochastic function record
 z = height of wind-velocity measurement
 z_r = roughness parameter (Section 2.7)
 α = coefficient of Voznessensky and Firsoff's (Section 6.6) spectrum, characteristic of sea irregularity (spectrum broadness)
 β = coefficient of Voznessensky and Firsoff's (Section 6.6) spectrum, characteristic of wave frequency

β = Jeffrey's sheltering coefficient (Section 2.1)	μ = molecular viscosity
β = wave age, i.e., the ratio c/U	μ^* = turbulent viscosity
γ = tangential-drag-force coefficient; $\gamma = C_d^*/2$	ν = kinematic viscosity μ/ρ
δ, Δ = an increment	ρ = water density
ϵ = phase angle	ρ' = air density
ϵ = parameter characteristic of a spectrum's broadness (Section 8.6)	τ = tangential drag
$\eta(t)$ = wave elevation; ordinates of wave record measured from mean level	τ = time lag in spectral analysis
θ = direction of wave-component propagation with respect to predominating direction	$\psi(x, y, t)$ = three-dimensional correlation function
θ_0 = spread of wave directions (Cox and Munk, Section 4.3)	ω = circular frequency = $2\pi/T = 2\pi f$
λ = wave length	σ = standard error
	σ = total wave slope
	σ_c = cross-wind wave slope
	σ_u = up/down wind-wave slope

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Code of Abbreviations

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 An. Met.—Annalen der Meteorologie (Hamburg).
 An. N. Y. Acad. Sc.—Annals of the New York Academy of Sciences.
 DTMB—David Taylor Model Basin.
 Deut. Hydr. Zeit.—Deutsche Hydrographische Zeitschrift.
 Hansa—Die "HANSA," Zeitschrift für Schiffahrt-Schiffbau-Hafen.
 JSTG—Jahrbuch der Schiffbautechnischen Gesellschaft.
 J. Aero/Sp. Sc.—Journal of the Aero/Space Sciences.
 J. Mar. Res.—Journal of Marine Research.
 J. Sc. Inst.—Journal of Scientific Instruments.
 J. Fluid Mech.—Journal of Fluid Mechanics.
 J. Met. Soc. Japan—Journal of the Meteorological Society of Japan.
 INA—Transactions of the Royal Institution of Naval Architects, London, England.
 NACA—National Advisory Committee for Aeronautics.
 NBS—National Bureau of Standards.
 NEC—Transactions of the North East Coast Institution of Engineers and Shipbuilders.
 NSMB—Proceedings, Symposium on the Behavior of Ships in a Seaway, 25th Anniversary of the Netherlands Model Basin, Wageningen, 1957, final two-volume edition.
 New Zeal. J. Sc. Tech.—New Zealand Journal of Science and Technology.
 Phil. Mag.—Philosophical Magazine.
 Phil. Trans. R. Soc. A—Philosophical Transactions of the Royal Society of London, Series A.
 Proc. Am. Soc. C. E.—Proceedings of the American Society of Civil Engineers.
 Proc. IRE—Proceedings of the Institute of Radio Engineers.
 Proc. R. Soc. A—Proceedings of the Royal Society, Series A, London.
 Q. J. R. Met. Soc.—Quarterly Journal of the Royal Meteorological Society.
 Symp. Autocor.—Symposium on Applications of Autocorrelation Analysis to Physical Problems, Woods

Hole, Massachusetts June 13–14, 1949, Office of Naval Research, Dept. of the Navy, Washington, D. C.
 SNAME—Transactions of the Society of Naval Architects and Marine Engineers, New York.
 Tr. Am. Geo. Un.—Transactions of the American Geophysical Union.
 WRH—Werft-Reederei-Hafen.
 ZAMM—Zeitschrift für Angewandte Mathematik und Mechanik.
 Z. Met.—Zeitschrift für Meteorologie.
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 ETT—Experimental Towing Tank (now Davidson Laboratory—DL), Stevens Institute of Technology, Hoboken, N. J.
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1 Introduction

Chapter 2 deals with definition and evaluation of hydrodynamic forces acting on the hull of an oscillating ship in waves. The oscillating motion of a ship will be discussed in detail in Chapter 3. However, the forces and motions are so closely interconnected that a complete separation of these two subjects is not possible, and a certain minimum information on motion has to be included in Chapter 2 as well.

The exposition given in Chapter 2, as indeed in all subsequent chapters, follows the policy outlined in the Introduction to the Monograph (pages iv and v). An attempt has been made at a critical summary of the existing state of the art. It is expected that the reader will be stimulated to further research by the realization of the scope and the shortcomings of present knowledge of the hydrodynamic forces acting on a ship oscillating in waves. A summary of suggestions for research will be provided at the end of the chapter.

Because of the close relationship between the subject matter of Chapters 2 and 3, the bibliography for both is placed at the end of Chapter 3. The reader is asked to refer to it whenever the reference gives only the year of publication, thus "Davidson and Schiff (1946)." When a reference is made to other chapters it is preceded by the chapter number; thus, "Pierson (I-1957)."

1.1 Forces Acting on a Body Oscillating in a Fluid.

A continuously changing pattern of water velocities relative to the hull is created when a ship oscillates in waves. By virtue of the Bernoulli theorem, these water velocities and their rates of change cause changes of the water pressure on the hull. These pressures, acting in various directions, always normal to elements of the hull surface, can be resolved along three axes, x , y , and z , and the components can be integrated over the entire area of the hull so as to give the total resultant force in each of these directions. The force components also can be multiplied by the distance to the center of gravity of a ship, and integrated to give the total moment about each axis. It has been found that once the detailed derivation has been carried out, the actual evaluation of the forces often can be accomplished by a much simpler procedure in terms of the body volume.

The actual mechanism of a ship oscillating along three axes—surging, side sway and heave, and rotating about three axes, rolling, pitching and yawing—can be complicated. Nevertheless, the basic concepts and terminology are defined in the same way as for a simple harmonic oscillator. A can buoy in heaving motion in

low, long waves is a good example of a simple forced oscillation. Its motion is described by a linear differential equation of the second order

$$a\ddot{z} + b\dot{z} + cz = F_0 \cos \omega t \quad (1)$$

Here the first term $a\ddot{z}$ denotes the forces connected with the acceleration d^2z/dt^2 , and the coefficient a is a mass. It is in reality the mass m of the buoy itself plus a certain imaginary water mass m_z , the acceleration of which gives a heaving force equal to the vertical resultant of all fluid pressures due to actual acceleration of water particles in many directions. This imaginary mass is known as "added mass" or "hydrodynamic mass" and the coefficient a is written as $m + m_z = m(1 + k_z)$, where k_z is the "coefficient of accession to inertia" in the vertical plane. The total mass represented by the coefficient a is known as "apparent mass" or "virtual mass."

The second term $b\dot{z}$ denotes the force proportional to the instantaneous vertical velocity dz/dt . The coefficient b is known as "damping coefficient" for a reason to be discussed shortly. In most cases it is assumed to be constant. In reality it is often not constant and in application to ship rolling, for instance, it often has been taken as depending on velocity squared \dot{z}^2 as well as on \dot{z} . However, a satisfactory description of many forms of oscillation in nature is given by the linear form of Equation (1).

The term cz is the force proportional to displacement, and is usually known as the "restoring force," while the coefficient c is often referred to as a "spring constant." This is a force exerted per unit of displacement z . In the present example of a can buoy, the constant c is the heaving force caused by a change of draft of one unit; i.e., 1 ft in the foot-pound system.

The term $F_0 \cos \omega t$ on the right-hand side of equation (1) is the "exciting force." In the present simple example, F_0 is the amplitude of the buoyant force due to wave height. In a ship's case it also will depend on water velocities.

In the forced motion with harmonic exciting force, equation (1), the motion, after sufficient time, is also a simple harmonic so that body position at any instant is

$$z = z_0 \cos (\omega t + \epsilon) \quad (2)$$

where ω is the circular frequency, and ϵ is the "phase lag angle." Term z_0 is the amplitude of motion defined in its relationship to the amplitude of exciting force F_0 by

$$z_0 = F_0 [(c - a\omega^2)^2 + b^2\omega^2]^{-1/2} \quad (3)$$

It is of interest to establish what work is done by an oscillating body on the fluid per cycle of oscillation. On the basis of equation (2):

$$\begin{aligned} z &= z_0 \cos(\omega t + \epsilon) \\ dz &= -\omega z_0 \sin(\omega t + \epsilon) dt \\ \dot{z} &= -\omega z_0 \sin(\omega t + \epsilon) \\ \ddot{z} &= -\omega^2 z_0 \cos(\omega t + \epsilon) \end{aligned} \quad (4)$$

and the work done, in the period T , by the acceleration forces

$$\begin{aligned} a \int_0^T \ddot{z} dz &= a\omega^3 z_0^2 \\ &\times \int_0^T \sin(\omega t + \epsilon) \cos(\omega t + \epsilon) dt = 0 \end{aligned} \quad (5)$$

by damping forces proportional to \dot{z}

$$\begin{aligned} b \int_0^T \dot{z} dz &= b\omega^2 z_0^2 \int_0^T \sin^2(\omega t + \epsilon) dt \\ &= b\omega^2 z_0^2 T/2 \end{aligned} \quad (6)$$

by restoring forces, proportional to z ,

$$\begin{aligned} c \int_0^T z dz &= -c\omega z_0^2 \\ &\times \int_0^T \sin(\omega t + \epsilon) \cos(\omega t + \epsilon) dt = 0 \end{aligned} \quad (7)$$

Thus, it is seen that the average amount of work done on a fluid by acceleration and by restoring forces is nil. A body does the work on the fluid during a half cycle, and the fluid does an equal amount of work on the body during another half cycle. Only damping forces do a net amount of work on the fluid, and therefore take the energy out of the body and dissipate it in the fluid. In an oscillation of a free body this causes gradual diminution of the amplitude of motion, from which the term "damping" has been derived. In a continued forced oscillation the energy necessary to maintain it is supplied by the exciting forces.

If the frequency of the oscillation is low enough, the phase lag is negligibly small and the displacement of a body z is in phase with the exciting force. Equations (4) show that the velocity \dot{z} is 90 deg out of phase and the acceleration \ddot{z} is 180 deg out of phase.

The forces caused by water pressures can be divided into two groups. The restoring force cz is caused by hydrostatic water pressures. The hydrodynamic forces $b\dot{z}$ and $m_z\ddot{z}$ result from the velocities and accelerations of water particles. These two forces are in reality two components of the resultant of all hydrodynamic (i.e., exclusive of hydrostatic) water pressures.

Confusion has occasionally resulted from the descriptive definitions of the damping force and the inertial (acceleration of the hydrodynamic mass) force given earlier. In the recent literature there has been, therefore, a tendency to define these forces merely as the out-of-phase (by 90 deg) and in phase (in reality 180 deg out-of-phase) components of the hydrodynamic force.

1.2 Order of Exposition. Equation (1) was introduced in order to define four categories of forces acting on an oscillating body, namely, inertial, damping, restor-

ing, and exciting. The following sections of Chapter 2 will be devoted to the evaluation of these forces by theoretical and experimental means. Theoretical evaluation of hydrodynamic forces in harmonic oscillations has been approached in three ways:

- Comparison with ellipsoids (in Section 2)
- Strip theory (in Sections 3, 4 and 5)
- Direct three-dimensional solution for mathematically defined ship forms (Section 6).

In Sections 7 and 8 the forces in transient (slamming) conditions will be discussed.

2 Estimates of Hydrodynamic Forces and Moments by Comparison With Ellipsoids

The problem of forces and moments exerted by a fluid on a body moving in it received the attention of hydrodynamicists at a very early date, and chapters on this subject are found in all major books on hydrodynamics (see Chapter 1: References C, pp. 353-393; D, pp. 160-201; F, pp. 464-485). The problem is usually formulated for a body moving within an infinite expanse of a fluid initially at rest and assumed to be nonviscous. Only the forces due to the fluid inertia can therefore be present.

The forces and moments acting on a body can be evaluated by two methods. In the first method the pressure p acting on each element of a body surface is computed by Bernoulli's theorem

$$p = p_0 + \rho \frac{\partial \phi}{\partial t} - \rho \frac{q^2}{2} \quad (8)$$

where ϕ is the velocity potential, and q is the local fluid velocity at the surface of the body, induced by its motion. By taking components of pressures p in the desired direction and integrating over the surface of the body, the total force is obtained.

The second method consists of expressing the rate of change of the kinetic energy contained in a volume of fluid between the body surface and an imaginary control surface taken at a sufficiently large distance from the body. The kinetic energy T is given by the expression (Lamb, I-D, p. 46)

$$T = \frac{1}{2} \rho \int_S \phi \frac{\partial \phi}{\partial n} dS, \quad (9)$$

where n denotes the outward normal and S the surface of a body over which the integral is taken. The force is then found by differentiation of the energy with respect to the body displacement; for instance, the force X in the direction of the x -axis is

$$X = \partial T / \partial x \quad (10)$$

In the application of either of the foregoing methods it is necessary to obtain the velocity potential ϕ . Also it is necessary to have the mathematical description of a body in order to formulate expressions for the directions of the normals, and to permit the integration over a sur-

face. The needed mathematical expressions reduce to tractable forms only for deeply submerged ellipsoids. Since the forces in this case are inertial, they can be expressed in terms of the "coefficients of accession to inertia" k , defined as

$$1 + k = \frac{\text{total inertia of a body floating in a fluid}}{\text{inertia of fluid displaced by body}} \quad (11)$$

In connection with the objectives of the present monograph, interest is concentrated on prolate ellipsoids in which the major semi-axis a is taken to coincide with the x -axis in which also the mean body-velocity vector \mathbf{U} lies. The minor semi-axes b and c (not necessarily equal) are then taken to coincide with the y and z -axes. The oscillatory motion of the body may include translations along any of the three axes and rotations about any of these axes. The coefficients of accession to inertia have different values for any of those motions, and the symbol k is supplemented by a suitable subscript. Treating the motions of an ellipsoid of revolution (a spheroid), Lamb (Chapter 1-D) designated by k_1 the coefficient of accession to inertia for accelerations along the major semi-axis a (i.e., x -direction), by k_2 that for accelerations along a minor axis, and by k' that for rotation about a minor axis. These designations were used (among others) by Davidson and Schiff (1946), Korvin-Kroukovsky and Jacobs (1957, also Appendix C to this monograph) and Macagno and Landweber (1958). It has been recommended¹ that symbols k_x , k_y , and k_z be used for translations along axes and k_{xx} , k_{yy} , and k_{zz} for rotation about axes indicated by subscripts. This notation was used by Weinblum and St. Denis (1950). While convenient for treatment of multicomponent motion of three-dimensional bodies this notation may be confusing in discussing two-dimensional flows in the strip theory of slender bodies. In this case it is customary to take the x -axis laterally in the plane of water and the y -axis vertically. In order to avoid confusion with the three-dimensional analysis the notation k_v and k_h can be used² for the coefficients of accession to inertia in the vertical and horizontal (lateral) directions. Here k_v is identical with k_2 of Appendix C.

A brief table of coefficients for spheroids will be found in Lamb (Chapter 1, D, p. 155). Curves of the coefficients of accession to inertia or to moment of inertia for various proportions of the ellipsoid axes can be found in Zahn (1929), Kochin, Kibel and Rose (Chapter 1, C, pp. 385-389), and Weinblum and St. Denis (1950).

Since the exact evaluation of the coefficients of accession to inertia of ship forms is practically impossible, it has been customary to estimate them by comparison with ellipsoids of similar length, beam and draft. A typical application of this method is found in the work of Weinblum and St. Denis.

¹ Minutes of the first meeting of the Nomenclature Task Group of the Seakeeping Panel, SNAME.

² Subscripts v and h were used by Landweber and de Macagno (1957).

In making these estimates for surface ships an assumption is introduced that the coefficients of accession to inertia, initially derived for a deeply submerged body, are still valid for a body floating on the surface. In other words, the effects of wavemaking on the free water surface are neglected. These effects have been investigated in the simpler "strip theory" to be discussed in the next section. It appears that, within the practical frequency range, the coefficient k_z for heaving oscillations of a floating body may be, on the average, 80 per cent of that computed by comparison with a deeply submerged ellipsoid.

It is clear that comparisons with ellipsoids are limited to investigations of ship motions of a general nature, in which only the over-all proportions are involved and the details of the hull form are not considered. In addition, the results are evidently applicable to investigation of the motion of a ship, but provide no information on distribution of forces along the length of a ship. Knowledge of this distribution is necessary in calculating the bending moments acting on a ship in waves.

Theories and computations made for ellipsoids have been important in bringing out certain trends or laws of action of hydrodynamic forces which are indicative of what can be expected in ships and submarines. As typical examples of this theoretical activity, the work of Havelock (1954, 1955, 1956) and Wigley (1953) can be cited.

3 Evaluation of Forces in Heaving and Pitching by Strip Theory

As has been mentioned earlier, solutions of three-dimensional hydrodynamic problems have been limited to ellipsoids, and are practically impossible when dealing with ships.³ The strip theory has been introduced in order to replace a three-dimensional hydrodynamic problem by a summation of two-dimensional ones. Using this method, solutions are possible for a much wider range of problems and actual hydrodynamic conditions connected with ship motions can be represented more completely. F. M. Lewis (1929) appears to be the first to apply this theory in connection with evaluation of hydrodynamic forces acting on a vibrating ship. Hazen and Nims (1940), St. Denis (1951), and St. Denis and Pierson (1953) used the strip theory in connection with the analysis of ship motions. This theory was described more explicitly later by Korvin-Kroukovsky (1955c) and Korvin-Kroukovsky and Jacobs (1957). Quoting from the latter work:

"Consider a ship moving with a constant forward velocity (U) (i.e., neglecting surging motion) with a train of regular waves of celerity (c). Assume the set of co-ordinate axes fixed in the undisturbed water surface, with the origin instantaneously located at the wave nodal point preceding the wave rise, as shown in Fig. 1 [here-with]. With increase in time t the axes remain fixed in space, so that the water surface rises and falls in relation

³ Solutions of hydrodynamic problems related to special mathematically defined ship forms will be discussed in Section 6.

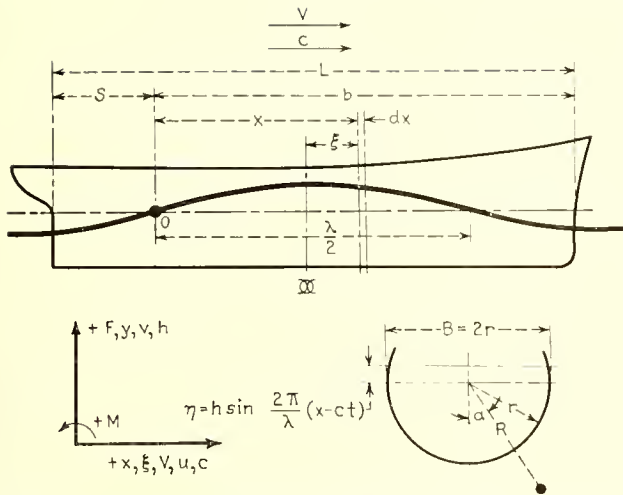


Fig. 1 Sketch illustrating notation used in connection with strip theory (from Korvin-Kroukovsky, 1955c)

to them. This vertical displacement at any instant and at any distance x is designated η . Imagine two control planes spaced dx apart at a distance x from the origin, and assume that the ship and water with orbital velocities of wave motion penetrate these control surfaces. Assume that the perturbation velocities due to the presence of the body are confined to the two-dimensional flow between control planes; i.e., neglect the fore-and-aft components of the perturbation velocities due to the body, as in the 'slender body theory' of aerodynamics. This form of analysis, also known as the 'strip method' or 'cross flow hypothesis' is thus an approximate one in the sense that a certain degree of interaction between adjacent sections is neglected."

In analyzing ship motions it is generally necessary to stipulate two systems of axes, one fixed in space and one fixed in the body. Thus, considering heaving and pitching of a ship, Korvin-Kroukovsky and Jacobs (1957)⁴ stipulated an x, y, z -system fixed in space (with the x - y -plane in the undisturbed water surface) and a ξ, η, ζ -system fixed in the ship. The location in the ship of the origin of the latter system is arbitrary, but the mathematical work is simplified considerably if the origin is placed at the center of gravity. A primary step in the strip method of analysis is to evaluate the hydrodynamic forces caused by the relative ship-wave vertical motion at a ship section located at a distance ξ from the origin. The vertical velocity of this section is the summation of the velocity components in heaving \dot{z} and in pitching $\xi\dot{\theta}$. When a ship is at a small angle of trim θ , the draft of ship sections, passing through the water slice dx , increases with time. This gives an added vertical velocity component θV .

After the forces acting on individual ship sections are evaluated, they are integrated over the ship length.

The integral forms used to obtain various coefficients for the equations of motion are given in Appendix C and are discussed in Chapter 3. Use of the sectional forces in computations of the hull bending moments are discussed in Chapter 5, by Jacobs (5-1958) and by Dalzell (5-1959).

The forces produced by water pressures on ship sections can be classified by their nature as inertial, damping, and displacement. They also can be classified by their cause as resulting from a ship's oscillation in smooth water or from wave action on a restrained ship. Kriloff (1896, 1898), considering only displacement forces, demonstrated that the total force acting on a ship in waves can be considered as the sum of these two components. Korvin-Kroukovsky and Jacobs (1957) demonstrated that this subdivision of forces also holds (within linear theory) when the pressures are generated by water acceleration. This is the direct consequence of the linear superposition of velocity potentials defining various water flows. It can be added here that the forces involved in pitching and heaving are caused primarily by potential flows, and water viscosity does not appear to be important in this connection.

The sectional inertial forces will be discussed in the following Section 3.1 and the damping forces⁵ in Section 3.2. The action of displacement forces in a ship oscillating in smooth water is obvious and needs no discussion. The displacement effect caused by waves will be brought out in the consideration of inertial forces, since the wave elevations are inseparably connected with water accelerations.

3.1 Inertial Forces Acting on a Body Oscillating in Smooth Water: 3.11 Conformal transformations.

The work on added mass in vertical oscillations most often referred to is that of F. M. Lewis (1929). Lewis assumed that water flow around a circular cylinder floating half immersed on the water surface is identical with that around a deeply immersed cylinder. Simple expressions for the latter are available in standard textbooks (Chapter 1, References C, D, and F). The added mass of a cylinder is found from these expressions to be equal to the mass of water displaced by it. The coefficient of accession to inertia is, therefore, unity. Lewis devised a conformal transformation by means of which a circle is transformed into ship-like sections of various beam/draft ratios and sectional coefficients. Water flows corresponding to these sections were derived and coefficients of accession to inertia were determined. In addition to Lewis' (1929) original work, the procedure was described (with various extensions) by Prohaska (1947), Wendel (1950), and Landweber and de Macagno (1957). The resultant relationships were also given by Grim (1956).⁶

The original half-immersed circle of radius r is defined in complex form

⁴ The mathematical part of this reference is included in this monograph as Appendix C.

⁵ It will be shown in Section 3.2 that damping forces are also of inertial origin.

⁶ An independent evaluation of added masses was also made by J. Lockwood Taylor (1930b).

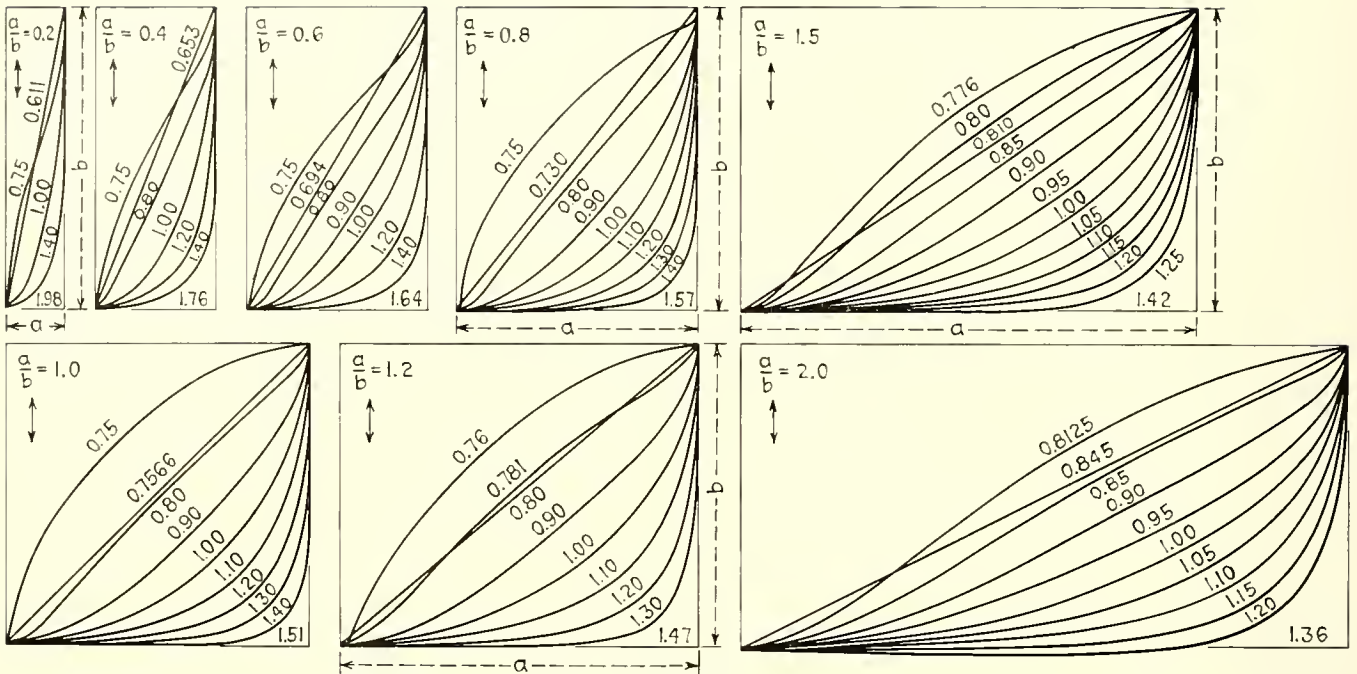


Fig. 2 Transverse sections and inertia coefficients obtained by Lewis (from Wendel, 1950)

$$\zeta = \xi + i\eta = re^{i\theta}$$

where θ is the angle which the radius vector makes with the water level. The transformed figure is described by F. M. Lewis as

$$z = x + iy = \zeta + \frac{a_1}{\zeta} + \frac{a_3}{\zeta^3} \quad (12)$$

$$= re^{i\theta} + \frac{a_1}{r} e^{-i\theta} + \frac{a_3}{r^3} e^{-3i\theta}$$

where a_1 and a_3 are real coefficients. After separation of real and imaginary parts, the co-ordinates x and y are given by parametric equations

$$\begin{aligned} x/r &= (1 + a_1) \cos \theta + a_3 \cos 3\theta \\ y/r &= (1 - a_1) \sin \theta - a_3 \sin 3\theta \end{aligned} \quad (13)$$

The half-beam b and the draft d of the resultant figure are related to the coefficients a_1 and a_3 by

$$\begin{aligned} b/r &= 1 + \frac{a_1}{r^2} + \frac{a_3}{r^4} \\ d/r &= 1 - \frac{a_1}{r^2} + \frac{a_3}{r^4} \end{aligned} \quad (14)$$

The section shapes resulting from this transformation are shown on Fig. 2. Each separate set of curves corresponds to a different b/d ratio (labelled on the figure as a/b). The curves are labelled by the values of the added mass coefficient C (defined later). The validity of this transformation is limited to the range of parameter values shown in Table 1. Fig. 3 shows the values of the

Table 1

(From Landweber and de Macagno, 1957)

d/b	$\beta = A/2bd^*$
0.6	0.412-0.93
0.8	0.353-0.942
1.0	0.294-0.957
1.4	0.379-0.937
1.8	0.425-0.925
2.5	0.471-0.914
5.0	0.530-0.898

* A is sectional area.

added mass coefficient C of Lewis forms as a function of the section coefficient β and the draft/beam ratio.

F. M. Lewis expressed the added mass in terms of the water mass enclosed within a semi-circular contour of radius b as the coefficient

$$C_v = \frac{\text{added mass}}{\pi \rho b^3 / 2} \quad (15)$$

For the ship sections obtained from the foregoing transformation the coefficient is evaluated as

$$C_v = \frac{(1 + a_1)^2 + 3a_3^2}{(1 + a_1 + a_3)^2} \quad (16)$$

The computed values of the coefficient C_v are shown in Fig. 3. The coefficient C_v is related to k_v the coefficient of accession to inertia by

$$k_v = C_v \pi B^3 / 8A \quad (17)$$

where B is the sectional beam, and A is the sectional area $k_v = C_v$ for a semi-circular profile.

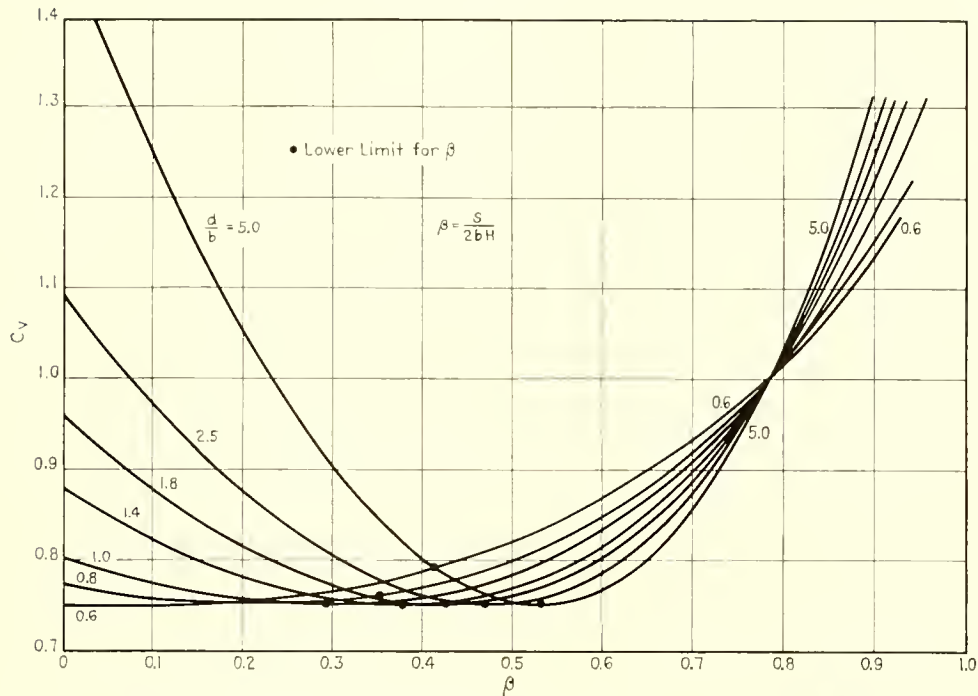


Fig. 3 Added mass coefficients for vertical motion (from Landweber and de Macagno, 1957)

Landweber and de Macagno (1957) have shown that F. M. Lewis' transformation is a particular case of a more general transformation form

$$z = \zeta + a_1/\zeta^m + a_2/\zeta^n \dots \quad (18)$$

in which the indices m , n , etc., are odd numbers in the case of symmetrical sections. Probaska (1947) computed the properties of sections with indices (m, n) of (1,5), (1,7), and (3,7).

The transformation from a circle, described in the foregoing, is not suitable for section forms with sharp edges. The water-flow pattern around polygonal forms and the resulting added masses can be obtained by means of the Schwartz-Christoffel transformation. A good description of this with examples of application will be found in Wendel (1950). Rectangles, rectangles with bilge keels, and rhombuses were analyzed by this method. The data on these will be found in Figs. 4 and 5.

Experiments, based on the analogy between the electric potential and the velocity potential of a fluid flow (Koch, 1933), give results identical in principle to those obtained from conformal transformations. They can be useful where conformal transformation becomes laborious, as for instance in the case of the added mass of a floating body in shallow water. They could also be useful in cases to which the available computational methods do not apply. A ship section composed of curves and equipped with bilge keels can be cited as an example, as can also typical sections in the stern portions of commercial ships. These sections are usually bounded by curved lines and possess either a large vertical

deadwood, or propeller-shaft bossings, or both. The hydrodynamic properties of such sections are not known, and their evaluation by means of electrical analogy can be recommended.

An extended discussion of added mass (neglecting free-surface effects) will be found on pages 417-441 of Volume II of "Hydrodynamics of Ship Design" by Harold E. Saunders published by The Society of Naval Architects and Marine Engineers, 1957. A listing of 54 references is included in this work.

3.12 Effect of the free water surface. The evaluation of the added masses in the foregoing was based on the assumption that the water flow about the submerged part of a floating profile is identical with the flow about a submerged profile consisting of the original one and its reflection in the water surface. This means that the wavemaking by an oscillating floating body was neglected. Subsequent mathematical work (Havelock, Ursell, Haskind) has shown that the foregoing assumption is valid for a high oscillation frequency. The values obtained on this basis are therefore directly applicable to ship vibrations, which was indeed the application intended by F. M. Lewis and Probaska.

A different situation is found at the low frequency of a ship's pitching and heaving in waves. Two wave systems are generated by these motions: The standing-wave system in proximity to the ship, and the progressive waves running away from the ship. The mean energy content of the first of these systems remains constant and defines the added mass. The progressive waves carry the energy away from the ship and represent the

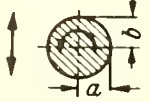
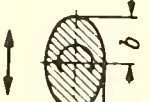
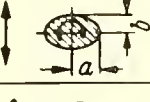
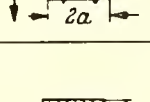
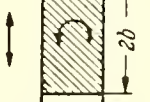
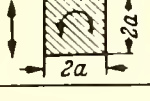
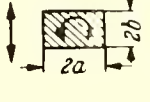
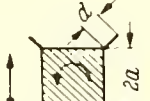
Cross-Sectional Forms		Hydrodynamic Mass per Unit Length	Inertia Coefficient $C = \frac{m''}{m''_{\text{circle}}} =$	Hydrodynamic Moment of Inertia per Unit Length	Inertia Coefficient for Rotation $D'' = \frac{J''}{J''_{\text{plate}}} =$
Direction of Motion	Axis of Rotation				
	$\frac{a}{b} = 1$ (circle)	$m'' = \pi \rho a^2$ [1]	1	$J'' = 0$	0
		$\left. \begin{array}{l} [1] \\ = \pi \rho a^2 \end{array} \right\}$	1	$[1]$ $= 0.125 \pi \rho (a^2 - b^2)^2$	$\left[\left(\frac{b}{a} \right)^2 - 1 \right]^2$
					
	Plate	$= \pi \rho a^2$ [1]	1	$= 0.125 \pi \rho a^4$ [1]	1
	$\frac{a}{b} = \frac{1}{2}$	$= 1.7 \pi \rho a^2$ [2]	1.7	$= 0.15 \pi \rho b^4$ [5]	$\left. \begin{array}{l} 1.2 \\ 1.18 \end{array} \right\} \text{ Referred to Plate of Width } 2b$
	$\frac{a}{b} = \frac{1}{5}$	$= 1.98 \pi \rho a^2$ [2]	1.98	$= 0.15 \pi \rho b^4$ [5]	
	$\frac{a}{b} = \frac{1}{10}$	$= 2.23 \pi \rho a^2$ [2]	2.23	$= 0.147 \pi \rho b^4$ [5]	
	Square	$= 1.51 \pi \rho a^2$ [2]	1.51	$= 0.234 \pi \rho a^4$ [3, 5]	1.872
	$\frac{a}{b} = 2$	$= 1.36 \pi \rho a^2$ [2]	1.36	$\left. \begin{array}{l} [5] \\ = 0.15 \pi \rho a^4 \end{array} \right\}$	$\left. \begin{array}{l} 1.2 \\ 1.18 \end{array} \right\} \text{ Referred to Plate of Width } 2a$
	$\frac{a}{b} = 5$	$= 1.21 \pi \rho a^2$ [2]	1.21		
	$\frac{a}{b} = 10$	$= 1.14 \pi \rho a^2$ [2]	1.14		
	$\frac{d}{a} = 0.05$	$= 1.61 \pi \rho a^2$ [5]	1.61	$= 0.31 \pi \rho a^4$ [5]	$\left. \begin{array}{l} 2.4 \\ 3.2 \\ 5.5 \end{array} \right\} \text{ Referred to Plate of Width } 2a$
	$\frac{d}{a} = 0.1$	$= 1.72 \pi \rho a^2$ [5]	1.72	$= 0.4 \pi \rho a^4$ [5]	
	$\frac{d}{a} = 0.25$	$= 2.19 \pi \rho a^2$ [5]	2.19	$= 0.69 \pi \rho a^4$ [5]	

Fig. 4 Tabulation of hydrodynamic masses, hydrodynamic moments of inertia, and inertia coefficients as calculated by [1] Lamb, [2] Lewis, [3] Proudman, [4] Weinblum, [5] Wendel, [6] determined experimentally (electrical analog) by Koch (from Wendel, 1950)

primary cause of damping. The standing-wave system is very complex and the mathematical solution for the added mass has so far been obtained only for a floating

circular cylinder (Ursell, 1949b, 1953, 1955).⁷ The

⁷ Three-dimensional solutions for special mathematical ship forms by Haskind and Hanaoka will be discussed in Section 6.

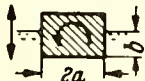
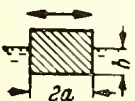
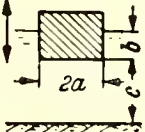
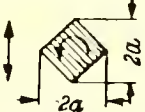
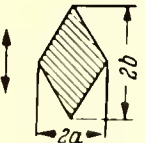
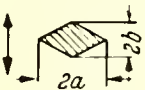
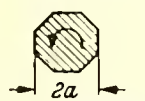
Cross-Sectional Forms		Hydrodynamic Mass per Unit Length	Inertia Coefficient $C = \frac{m''}{m''_{\text{circle}}} =$	Hydrodynamic Moment of Inertia per Unit Length	Inertia Coefficient for Rotation $D'' = \frac{J''}{J''_{\text{plate}}} =$
Direction of Motion	Axis of Rotation				
	$\frac{a}{b} = 1$	^[2] $= 0,75 \pi \rho a^2$ $= \frac{1}{2} \text{ square}$	0.75	^[5] $= 0.117 \pi \rho a^4$	0.936
	$\frac{a}{b} = 1$	^[5] $= 0,25 \pi \rho a^2$	0.25		
	$\frac{e}{b} = \infty$	^[5] $= 0.755 \pi \rho a^2$	0.75		
	$\frac{e}{b} = 2.6$	^[5] $= 0.83 \pi \rho a^2$	0.83		
	$\frac{e}{b} = 1.8$	^[5] $= 0.89 \pi \rho a^2$	0.89		
	$\frac{e}{b} = 1.5$	^[6] $\approx 1 \pi \rho a^2$	≈ 1		
	$\frac{e}{b} = \frac{1}{2}$	^[6] $\approx 1.35 \pi \rho a^2$	≈ 1.35		
	$\frac{e}{b} = \frac{1}{4}$	^[6] $\approx 2 \pi \rho a^2$	≈ 2		
		^[2] $= 0.76 \pi \rho a^2$	0.76	^[5] $= 0.059 \pi \rho a^4$	0.47
	$\frac{a}{b} = \frac{1}{2}$	^[2] $= 0.67 \pi \rho a^2$	0.67		
	$\frac{a}{b} = \frac{1}{5}$	^[2] $= 0.61 \pi \rho a^2$	0.61		
	$\frac{a}{b} = 2$	^[2] $= 0.85 \pi \rho a^2$	0.85		
	Regular Octagon			^[4] $\approx 0.055 \pi \rho a^4$	0.44

Fig. 5 Tabulation of hydrodynamic masses, hydrodynamic moments of inertia, and inertia coefficients as calculated by [1] Lamb, [2] Lewis, [3] Proudman, [4] Weinblum, [5] Wendel, [6] determined experimentally (electrical analog) by Koch (from Wendel, 1950)

added-mass coefficient was expressed as a function of a nondimensional frequency parameter $\omega^2 r / \pi g$. The results can be conveniently interpreted as a correction coefficient⁸

$$k_4 = \frac{\text{added mass of a body floating on water surface}}{\text{added mass of a body deeply submerged}} \quad (19)$$

⁸ This correction coefficient was first used in ship-motion analysis by B. V. Korvin-Kroukovsky (1955c). Having assigned (following Lamb) the notation $k_{1,2,3}$ to the coefficient of accession

to inertia of a submerged body in three directions, he used the notation k_4 for the added mass correction of a body floating on the water surface.

The values of coefficient k_4 , computed by Ursell for a floating cylinder, are listed in Table 2.

Table 2
(From Ursell, 1955)

$\omega^2 r / \pi g$	k_4
$1/6$	0.632
$1/4$	0.592
$1/2$	0.673
$2/3$	0.738
$3/4$	0.762
1	0.818
$5/4$	0.859
$3/2$	0.883

NOTE: For very high-frequency parameters the coefficient k_4 asymptotically approaches unity.

In the absence of data for other ship sections, Korvin-Kroukovsky (1955c) and Korvin-Kroukovsky and Jacobs (1957) have assumed that the coefficient k_4 , initially computed by Ursell for a circular cylinder, will apply to other profiles. Additional theoretical research is evidently needed in order to provide values of k_4 for F. M. Lewis' and other ship sections.

Attention should be called to the particular surface effect occurring in the case of sections with inclined sides. Such inclinations exist in the bow sections of V-form ships and are particularly pronounced in stern sections of most ships. When such a section penetrates the water surface, the adjacent water level rises and the wetted beam becomes greater than the one indicated by the intersection with the undisturbed water level. This causes an increase in added mass and probably in damping forces. A solution of this problem was given by Wagner⁹ for the asymptotic case of an impact in which gravity forces are neglected; i.e., the instantaneous water rise is taken into account but not the subsequent wave-making. In Wagner's solution, the effect of a deep draft in conjunction with a V-section has not been considered. J. D. Pierson (1950, 1951) used a computational method (initially suggested by Wagner) in which the draft of a V-section is included, subject to the gravity-free assumption. No information is available on the water flow, and the added masses and damping connected with it, for the water-surface penetration by a wedge at the frequencies of a ship's heaving and pitching in waves. Also, the available impact theories treat only the hydrodynamic force acting on a body when it penetrates the water. The theory of ship motions and bending moments requires also knowledge of the forces during body emergence from the water. The added masses involved in these motions will evidently be functions of oscillation frequency. Research in this field is needed.

3.13 Three-dimensional effects. In the evaluation of hydrodynamic forces by the strip theory, the initial assumption is made that the water flow at each strip is two-dimensional and is not influenced by the adjacent strips. Subsequently, it is desired to verify this assumption and, if practical, to establish a correction fac-

tor for the deviation of the physical conditions from the initially assumed ones. This problem can be considered in connection with three applications:

a) A body vibrating with various numbers of nodal points.

b) A body of a certain L/B ratio pitching and heaving, in smooth water.

c) A body subjected to waves.

The first problem (a) was treated by F. M. Lewis (1929) and J. Lockwood Taylor (1930b) with varying results. Macagno and Landweber (1958) have investigated these solutions and demonstrated that the results are strongly affected by the completeness with which the shear and flexural deflections of a body are described.

The second problem (b) has apparently received no attention. The third problem (c) was solved for a spheroid under waves by Havelock (1954) and Cummins (1954a, b) using advanced mathematical methods, and by Korvin-Kroukovsky (1955b) using the strip theory. The agreement between calculational methods was satisfactory (as shown in Figs. 15 and 16), and apparently no correction for three-dimensional effect is needed for the length/beam ratios normally used in ships and for waves of length approximately equal to a ship's length. This conclusion applies, however, to the coefficient of accession to inertia k_z for the entire body in the analysis of body motions. Three-dimensional effects, in all probability, do affect the distribution of hydrodynamic forces along a ship; i.e., the k_x -values for individual strips. These effects are, therefore, significant in the analysis of ship bending moments and research toward their evaluation is recommended.

3.14 Inertial forces caused by waves. It appears that satisfactory estimates of forces exerted by waves on a submerged body can be made considering inertial forces alone and neglecting viscous forces. This means that hydrodynamic-force components in phase with the waves can be expressed in terms of added mass. The added mass is expressed, in turn, in terms of body displacement. In a strip theory the displacement is taken per unit length. In considering the wave action on a body it is necessary to remember that a velocity gradient and a corresponding pressure gradient exist in waves.

The force acting on a small submerged body in long waves can be calculated most conveniently on the basis of this gradient and the body's volume. The force exerted by a hydrostatic pressure gradient is equal to the product of the gradient and the body's volume. G. I. Taylor (1928) showed that this relationship is modified when the pressure gradient in a fluid results from the acceleration of fluid particles. The relationship becomes

$$\text{Force} = (1 + k) \times (\text{pressure gradient}) \times (\text{body volume}) \quad (20)$$

where k is the coefficient of accession to inertia. The factor $(1 + k)$ is the result of modification of an accelerating fluid flow by the presence of a body.¹⁰

⁹ To be discussed in greater detail in Section 7 on slamming.

¹⁰ Forces exerted on a body by fluid accelerations were also investigated by Tollmien (1938).

In estimating the force acting on a body by the strip theory, the foregoing relationship is applied to each section using the appropriate value of the coefficient k .

A detailed derivation of the wave-caused force by means of surface-pressure integration was given by Korvin-Kroukovsky and Jacobs (1957) and is reprinted in Appendix C. In the derivation carried out for a semi-circular ship section, neglecting surface effects, the product of the sectional volume and the mean pressure gradient was found to be multiplied by 2. Since $k_v = 1$ for a semi-circular section, the factor of 2 was interpreted as $1 + k_v$ on the basis of G. I. Taylor's result. Grim (1957c) has confirmed this intuitive conclusion by application of F. M. Lewis' transformation. To correct for the surface effects neglected in the formal analysis, Korvin-Kroukovsky and Jacobs (1957) interpreted k_v as $k_1 k_2$ in evaluating the force exerted by the vertical wave pressure gradient on a surface ship. The calculated wave forces on a ship's model were confirmed by a towing tank test (Appendix 2 to Korvin-Kroukovsky, 1955c).

Attention should be called to the fact that the direction of the pressure gradient is such that the vertical force is acting downward on a submerged body under the wave crest, and upward under a trough. In a body floating on the water surface these pressure gradient (or inertial) forces are subtracted from the displacement force caused by water-surface rise in waves. The net force is thereby considerably reduced.

It is often convenient to think of water acceleration in waves as algebraically added to the acceleration of gravity. The water at wave crests appears then to be lighter and at wave troughs heavier than normal.

This modification of the effective weight of water in waves is often referred to as the "Smith effect," since attention was called to it by Smith (1883) in connection with ship bending-moment evaluation. Estimation of the wave forces acting on a ship by the buoyancy forces modified by the Smith effect is referred to as the "Froude-Kriloff hypothesis." The effect of the ship in disturbing waves is neglected in this case; i.e., the added term k in equation (20) is not taken into account. Since its inclusion is a simple procedure, there is no justification for neglecting it in the future.

3.15 Experimental data on inertial forces. Very few experimental data are available on added masses, and these, while confirming the general ideas outlined in the previous paragraphs, do not provide exact information. Experiments have been made for the following cases:

- a) Deeply submerged prisms and cylinders.
- b) Prisms oscillating on the free water surface.
- c) Ship forms oscillating on the water surface.
- d) Restrained ship forms and other bodies acted upon by waves.

1 Deeply submerged prisms and cylinders. Tests in the first category (a) are of interest for confirmation of the classical theory. The reasonableness of neglecting viscosity is the particular assumption to be verified. The frequency and amplitude of oscillations would be

irrelevant if water were a truly nonviscous fluid. The existence of a small viscosity, however, may cause eddy-making in certain experimental conditions, particularly in the case of a body with sharp edges. In such a case scale relationships may become significant.

Moullin and Browne (1928) experimented with two-node vibrations of flat steel bars submerged in water. The bars were from $1/4$ to 1 in. thick, 2 in. wide, and 78 in. long, so that three-dimensional effects probably were insignificant. The vibrations were excited by an electromagnet, and the added mass was obtained by comparison of the resonant frequencies in air and in water. It was concluded that the added mass is equal to the water mass of the cylinder circumscribing the rectangular profile of the bar. This is in agreement with the theoretically indicated added mass of a thin plate considering the expected increase with thickness of the rectangular section.

2 Prisms oscillating on the water surface. Moullin and Browne (1928) also experimented with bars $1/2$ in. thick and 3 in. wide, set on edge and partially submerged. They concluded that the added mass is independent of the vibration frequency. This conclusion is in agreement with theoretical expectations for high frequencies. Browne, Moullin and Perkins (1930) tested the vertical vibrations of rectangular and triangular prisms partly immersed in water. The prisms were attached to a flat steel spring and were vibrated by an electromagnet. A $6 \times 6 \times 54$ -in. prism vibrated at a frequency of about 15 cps. This frequency corresponds approximately (to scale) to the usual two-node frequency of ship vibrations. The theoretical added mass (for a submerged double profile) was computed by a Schwartz-Christoffel transformation. The experimentally determined added mass was found to be about 90 per cent of the theoretical one. Experiments were made with various lengths of prisms and the authors stated that, above a length/beam ratio of 4, the added mass was independent of the length. Todd (1933), in applying these and Lewis' (1929) results to ship-vibration analysis, attributed the reduction in added mass to the effect of the length/beam ratio.

It should be emphasized that the frequencies in the experiments just outlined correspond to ship-vibration frequencies. These are about 10 times as much as the usual frequency of a ship's pitching in head seas. Theory (Section 3.12) indicates that at lower frequencies a pronounced dependence of added mass on frequency can be expected.

Prohaska (1947) reported on oscillation tests of prisms of several profiles partly submerged in water. The experimental data indicated added-mass values about 90 per cent of the theoretical ones computed for the submerged double profiles. Unfortunately no information was furnished as to the frequency of oscillations. The description indicates that the test apparatus was of the same type used by Dimpker (1934) and Holstein (1936). With such an apparatus, the high frequency of Moullin's experiments can hardly be expected. Without information on the frequency, Prohaska's tests can be ac-

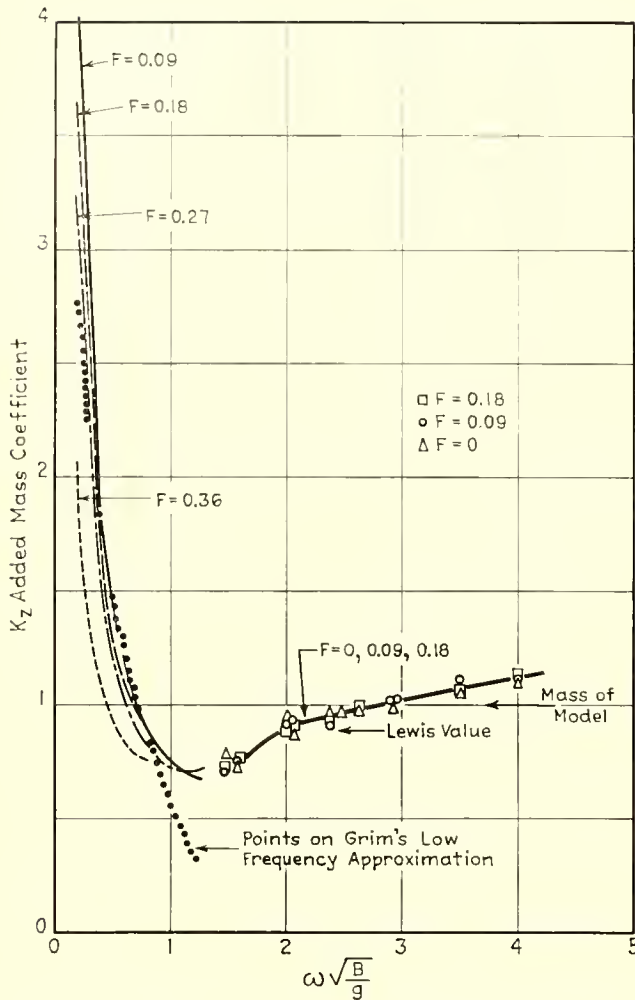


Fig. 6 Variation of added mass coefficient with frequency (from Golovato, 1957)

cepted only as a rough confirmation of the order of magnitude of the added masses. The roughly indicated 10 per cent reduction in added masses is clearly caused by the surface effects, since Prohaska's models spanned the width of the test tank and three-dimensional effects were absent.

Dimpker's (1934) and Holstein's (1936) tests were made at frequencies in the range estimated for ship motions in waves. These tests will be discussed in greater detail later in connection with damping (in Section 3.2) since damping forces as well as added masses were determined. Holstein's measurements of added masses appear to be too erratic to be useful. Wendel (1950), estimated the experimental errors in these measurements and demonstrated that added masses are considerably smaller than indicated by the submerged-prism theory. Dimpker, like Holstein, made tests at a series of frequencies governed by the stiffness of the retaining springs. He did not publish information on frequencies directly, but on spring stiffnesses. The data given in

the published paper do not appear to be sufficient to calculate the frequencies.

To summarize the results on the oscillation of partially submerged prisms: None of the tests made so far gives sufficient information on the frequency of oscillations to permit evaluation of the added mass versus frequency relationship. The tests have generally indicated that experimentally measured added-mass coefficients of bodies on the water surface are smaller than those for deeply submerged prisms. Evidently, additional experimental research is needed. All of the tests described in the foregoing were made in small tanks and it can be questioned whether the test data were not affected by wave reflections from tank ends. While wave-absorbing beaches have been used in towing tanks for many years, it was not realized until recently how difficult it is to prevent wave reflections.

In the work just described a strictly pragmatical approach was taken. Reference should be made to Weinblum (1952) and Keulegan and Carpenter (1956) for the less evident aspects of the inertial force and added-mass concepts. In particular, for bodies at the water surface the hydrodynamic force is connected with wave formation. It depends therefore not only on instantaneous conditions but on the past history of motions as well. Added mass becomes a definite concept only when correlated with a definite type of motion. The added masses in harmonic oscillation are not necessarily identical with the added masses in, for instance, uniform acceleration of a body. In the tests of Dimpker, Holstein, and Prohaska, the added masses were derived from the natural period of decaying oscillations. It can be questioned whether added masses so obtained are identical with those occurring in sustained harmonic oscillations.

3 Ship forms oscillating on the water surface. Golovato (1957*a,b*) reported on experiments with a harmonically heaving ship model restrained from pitching.¹¹ The model had lines composed of parabolic arcs, following Weinblum (1953), and had a prismatic coefficient of 0.655. The inertial and damping forces in heaving were calculated from the amplitude and phase lag of the motion records as compared with the records of the harmonic exciting force. Fig. 6 shows the coefficient of accession to inertia k_z plotted versus nondimensional frequency $\omega(B/g)^{1/2}$. A horizontal arrow at about $k_z = 0.93$ shows the value calculated by using F. M. Lewis' (1929) data; i.e., neglecting surface wave effects. The curve shown by heavy dots is Grim's (1953 *a,b*) asymptotic evaluation of the added mass for low frequencies. The experimental data at low frequencies are somewhat uncertain because of model interference with waves reflected from the sides of the towing tank. At higher frequencies the coefficient k_z is shown to be independent of the Froude number.

Fig. 6 covers a wide range of frequencies and the picture may be misleading unless the range important in

¹¹ The author understands that similar experiments also were made with pitching oscillations, but the results have not yet been published.

ship operations is kept in mind. Data on the pitching-oscillation frequencies of several ships will be found in Korvin-Kroukovsky and Jacobs (1957). For the Series 60 model of 0.60 block coefficient in head waves of $\lambda/L = 1$, the parameter $\omega(B/g)^{1/2}$ varies from 0.9 to 1.5 at ship speeds from zero to the maximum expected in smooth water. At synchronism in pitching the parameter is equal to 1.25. This narrow range of frequency parameters straddles the minimum of the curve of added mass coefficients in Fig. 6.

The reader will find it instructive to plot the Ursell data for the circular cylinder from Table 2 on Fig. 6, remembering that $r = B/2$. Both k_2 -curves plotted as a function of $\omega(B/g)^{1/2}$ are similar in form but the curve shown for Golovato's model is seen to be considerably above Ursell's curve. It should be noted that Golovato's curve does not asymptotically approach the F. M. Lewis value but is directed much higher. This raises the question of experimental reliability or perhaps the presence of physical features not accounted for by the theory. Reference to Keulegan and Carpenter (1956) indicates that the added mass may have been increased on the up-stroke of the oscillator by the separation of the water flow. If so, the data would be subject to the degree of model roughness and to a scale effect.

Gerritsma (1957c, d) tested a Series 60, 0.60 block coefficient model, 8 ft long, by subjecting it to forced oscillations in heaving and pitching alternately. Fig. 7 shows a comparison of measured and calculated virtual masses and virtual moments of inertia; i.e., ship masses plus added water masses. The calculated masses were taken from Korvin-Kroukovsky and Jacobs (1957), and are based on the strip integration of the product of F. M. Lewis' k_2 coefficients and the surface-effect correction coefficient k_4 based on Ursell's data. The discrepancy between measured and calculated data is small, and is in the right direction. In the Series 60 model the after-body ship sections have large inclinations to the water surface, and the surface effect caused by these inclinations was not taken into account. A correction for this effect (not known at present, see Section 3.12) can be expected to increase the calculated added masses.

It is gratifying to see that the discrepancies in virtual masses in heaving and in virtual moments of inertia in pitching are similar. This indicates that the three-dimensional effect is not important in added-mass evaluation.

To summarize the present section: Gerritsma's tests show a satisfactory agreement between added masses as measured and as calculated by the strip theory using the product of Lewis' and Ursell's coefficients, $k_2 k_4$. The need for an additional correction for the effect of inclined sides is indicated, and a correction for three-dimensional effect may be included in the future. However, the results of the motion analysis of usual ship forms would not be significantly affected by it. Golovato's tests on the idealized ship model show a greater value of the added mass than would be indicated by the method of calculation just described. The failure of the data to approach

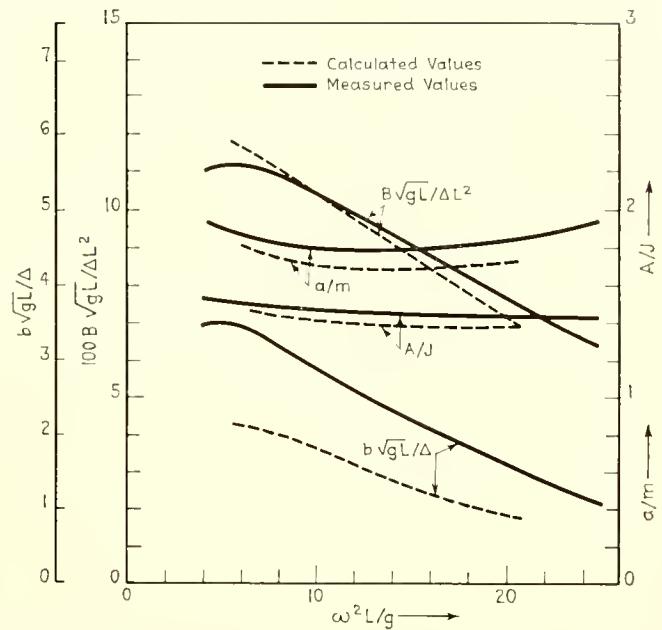


Fig. 7 Comparison between calculated and measured values of A , B , a , b , for series 60, $C_B = 0.60$ hull form (from Gerritsma, 1957d)

Lewis' value asymptotically at high frequencies raises suspicion and calls for added investigation. Since only two investigations have been reported, further research is evidently needed.¹²

4 Restained ship forms and other bodies subjected to wave action. Three references can be cited in connection with this subject: Keulegan and Carpenter (1956), Rechlin, Steele and Scales (1957), and Korvin-Kroukovsky (1955c, Appendix 2). A study of the first two, although they are primarily concerned with the forces acting on offshore structures in shallow water, should be fruitful to a researcher in naval architectural problems. The third reference appears to be the only work concerned directly with the forces acting on ships.¹³

The heaving force, pitching moment, and drag force exerted by waves on a ship model were measured. A Series 60, 0.60 block coefficient model, 5 ft long, was restrained from heaving, pitching and surging by dynamometers attached at 0.25 and 0.75 of the model's length. Tests were made in regular waves 60 in. long (i.e. $\lambda/L = 1$) and 1.5 in. high at six speeds of advance, starting with zero. Fig. 8 is a comparison of test data with calculations made by Korvin-Kroukovsky and Jacobs (1957) using strip theory and added-mass coefficient

¹² Minutes of the S-3 Panel of the SNAME indicate that such research is in progress at the Colorado State University under the guidance of Prof. E. F. Schulz. In this program added masses and damping forces are measured on individual sections of a segmented ship model so that the distribution of forces along the body length will be obtained.

¹³ Additional measurements of the wave-caused forces recently were published by Gerritsma (1958, 1960).

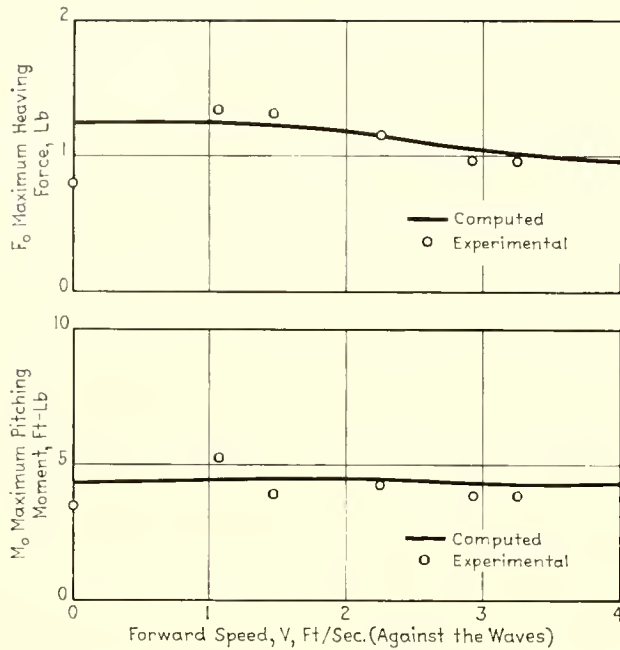


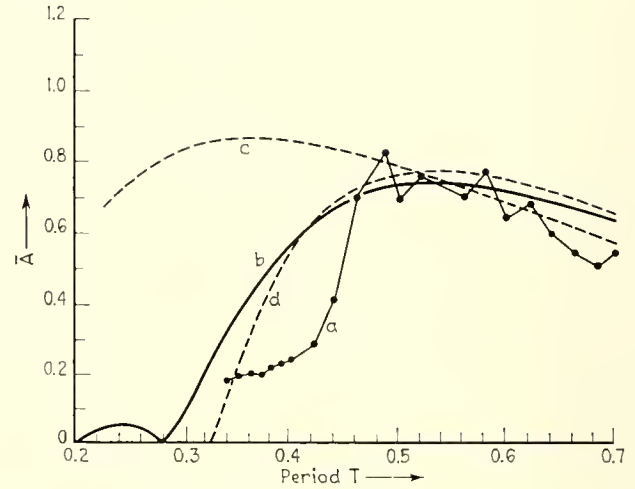
Fig. 8 Comparison of computed and experimentally measured exciting force and moment amplitudes for ETT Model 1445 in waves 5 ft by 1.5 in. (from Korvin-Kroukovsky, 1957)

$k_2 k_3$.¹⁴ A good agreement is demonstrated except at zero model speed. It is possible that at zero speed the reflections of model-radiated waves from towing-tank walls was affecting the test data. The influence of these reflections was not recognized at the time of the tests, but has since been found important by Golovato and Gerritsma.

While Fig. 8 appears to provide a satisfactory confirmation of the calculational procedure, it should be remembered that it is only the result of a single project of small scope. Additional tests by other investigators with several model forms and different wave lengths and heights are desirable.

The tests of Korvin-Kroukovsky just described, as well as those of Golovato and Gerritsma point to difficulties caused by wave reflections at low model speeds. Tests in wide (maneuvering) tanks are therefore recommended.

Attention should be called to the fact that the hydrodynamic force caused by waves is in composition like the force caused by ship oscillations: i.e. it has force components proportional to acceleration (inertial), to velocity and to displacement. In the analysis of ship motions, Korvin-Kroukovsky and Jacobs (1957) neglected the wave-force component proportional to velocity, since this component became very small after integration of sectional forces over a ship's length. This component cannot be neglected, however, for individual sections when the force distribution along the length is



(a)—Prism 12 cm wide, 5 cm draft—Holstein's experimental data
(b)—Same computed by source distribution along the bottom
(c)—Semi-cylinder of 6 cm radius—on basis of Ursell's (1949) computations
(d)—Semi-cylinder computed by source distribution over the surface

Fig. 9 Ratio, \bar{A} , of wave amplitude to amplitude of heaving motion of cylinder and of rectangular prism of same width and sectional area (from Korvin-Kroukovsky, 1955a)

important, as in the calculation of hull bending moments (Jacobs, 5-1958). Therefore, research planning must provide for the measurement of the distribution of velocity-proportional forces as well as the inertial ones.

3.2 Damping Forces. The work of Grim (1953) is particularly valuable in that it contains a comprehensive set of curves for estimation of damping. These curves are of \bar{A} , the ratio of the progressive waves' amplitude to the amplitude of heaving oscillation, as a function of nondimensional frequency $\omega^2 B/2g$ for Lewis-type sections. From the values of \bar{A} , the damping force coefficients $N(\xi)$ per unit length can be computed readily (Havelock, 1942b; St. Denis, 1951) by

$$N(\xi) = \rho g^2 \bar{A}^2 / \omega_e^3 \quad (21)$$

One of the drawbacks to following Grim's study is the extensive interpolation needed to obtain the values of \bar{A} and the damping-force coefficients for various ship sections. A more convenient approximate evaluation of damping had been developed earlier by Holstein (1936, 1937a, b) and Havelock (1942). Their method is based on the fact that formation of surface waves by a single pulsating source at a depth f can be evaluated as a preliminary step. The solution gives two types of wave systems, the standing waves and the progressive waves. The expression representing a progressive-wave system at a great distance from a body takes on a simple form. The action of a prismatic body in heaving oscillation is replaced by a series of pulsating sources on the body surface, and the total wave formation is obtained by integration of the effects of separate sources over the

¹⁴ The method of calculation is described in Appendix C.

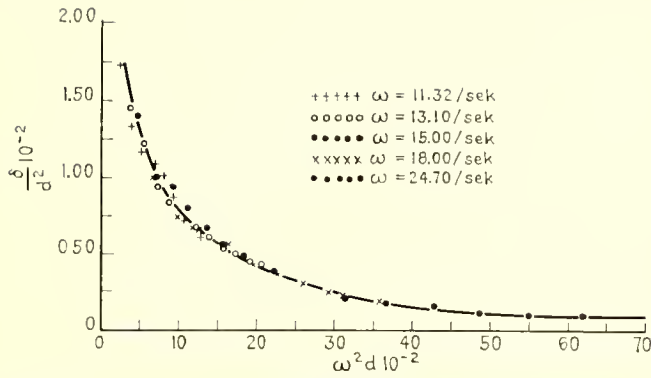


Fig. 10 Variation of logarithmic decrement, δ , with frequency, ω , in damping of heaving oscillations of 60-deg wedge at submergence d (from Dimpker, 1934)

contour. Havelock suggested that a ship section can be replaced by a rectangular one of draft f corresponding to the mean draft of the section; i.e., $f = A/B$, with sources distributed along the bottom. The resultant expression for the ratio \bar{A} is

$$\bar{A} = 2e^{-k_0 f} \sin(k_0 y) \quad (22)$$

where¹⁵ $k_0 = \omega^2/g$ and y is the half beam of a ship section under consideration. Fig. 9 taken from Korvin-Kroukovsky (1955a) gives a comparison of the ratio \bar{A} computed by three methods: As given for a semi-cylinder by Ursell (1949) (this agrees with Grim's values for a semi-cylinder); as computed by a source distribution over the contour; and as computed by a source distribution over the bottom of a rectangle of the same sectional area. The results of all methods are in reasonably good agreement at low frequencies, but differ considerably at high frequencies. Fortunately, the frequency of oscillation at synchronism of normal ships is generally in the region in which the disparity is not excessive, and both Havelock's and Grim's damping coefficients have been used with reasonable success. It should be remembered that damping is most important in the evaluation of motion amplitudes near synchronism, and that, at frequencies widely different from synchronism, large errors in estimated damping have relatively little effect on the amplitude. The effect of the damping on the phase lag of motions is, however, most pronounced at frequencies different from the synchronous one.

3.21 Experimental verification of the sectional damping coefficients. The complexity of advanced forms of the solution of the foregoing problem, such as Ursell's and Grim's, necessitates adopting various approximations, the effect of which is difficult to appraise. Therefore, experimental verification is desirable. In Holstein's and Havelock's method of calculation, no investigation is made of boundary conditions at a body, particularly as

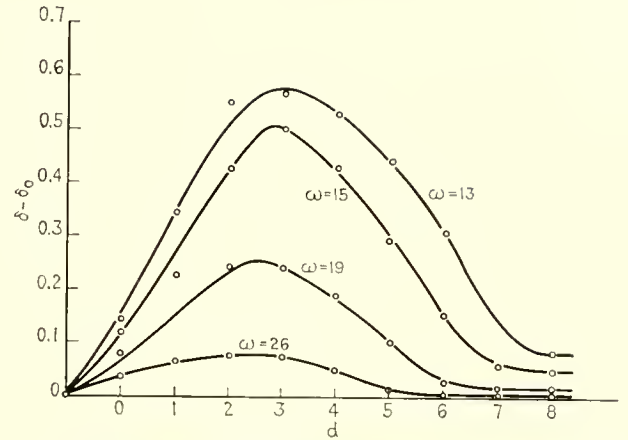


Fig. 11 Dependence of net logarithmic decrement, $\delta - \delta_0$, on submergence, d , with frequency, ω , as parameter for a cylinder 10 cm diam (from Dimpker, 1934). δ is the decrement measured in water, δ_0 is decrement measured in air in preliminary calibration

these are modified by the wave formation. Acceptance of this method depends entirely on successful experimental verification. Holstein (1936) made experiments to verify his theory. These were limited, however, to a rectangular prism varying in degrees of initial immersion, and were made in a small test tank 0.70 m (2.3 ft) wide by 3 m (10 ft) long.

The values of \bar{A} were established by comparison of directly observed wave amplitudes with the amplitudes (half strokes) of the heaving prisms. The results of a large number of experiments appear to be consistent, thus inspiring confidence. The small size of the tank, however, makes the data questionable. It should be remembered that to evaluate \bar{A} the wave amplitudes should be measured far enough from a body for the progressive wave system to be completely free from the standing waves. Furthermore, one must be certain that the progressive waves are not contaminated by reflection from the test-tank ends. These aspects of the test are not discussed sufficiently by Holstein, and, in view of the shortness of the test tank, they may be suspected as having affected the results. The use of a rectangular prism is also questionable, since a certain disturbance can emanate from its sharp edges. This effect is not provided for in the theory.

In his experiments Holstein also attempted to determine virtual masses, but the resultant data were too erratic to be useful.

Dimpker (1934) published data on experiments with a 60-deg wedge and a cylinder with various degrees of immersion. The wedge was tested with an initial immersion from 0 to 12 cm, and the 10-cm-diam cylinder with an initial immersion varying from 0 to 8 cm. The floating body was connected by springs to a motor-driven eccentric, so that either free or forced oscillations could be investigated. Only the free oscillations were dis-

¹⁵ It is necessary to distinguish between the frequency ω of the oncoming waves and the frequency ω_e of the wave encounter which is also the frequency of the ship-radiated waves.

cussed in the published paper.¹⁶ The oscillatory motions of the model were recorded on a rotating drum. The damping was defined by the logarithmic decrement δ as a function of the frequency ω of the oscillation and the mean submergence d .

For a 60-deg prism the mean submergence (over the oscillating cycle) is equal to the beam at mean waterline. Dimpker showed that data for tests at varying frequencies and submersions collapsed into a single curve when plotted as δ/d^2 versus $\omega^2 d$. These amplitude and frequency parameters were initially defined in a non-dimensional form. After the constant quantities, such as the mass involved, the acceleration of gravity g and the water density ρ were omitted, the parameters took the form indicated in the foregoing. The resultant curve is reproduced on Fig. 10. Data for the cylinder are given in Fig. 11. In this case the immersed shape varies with the draft d and it is not possible to make a generalized plot. It is interesting to note that the maximum damping occurs at an immersion of about 2.5 cm; i.e., half-radius. The decrease of damping with further immersion is in agreement with the general tendency shown by the theories of Holstein and Havelock (increase of f in equation 22).

Dimpker (1934) evaluated the virtual masses for a wedge and a cylinder on the basis of changes in the natural period resulting from changes of immersion and frequency. The frequencies were given in terms of spring constants, and the significance of results cannot be seen readily since insufficient data were given for recalculation.

Unfortunately, the experiments of Holstein and Dimpker appear to be the only published data in direct verification of the theory in regard to sectional damping coefficients. Verification of other theoretical methods is indirect. This consists of calculating ship motions using the coefficients evaluated on the basis of the previously mentioned methods and of accepting the successful motion prediction as justification for the method of calculation. However, it is far from being a reliable verification in view of the number of steps involved and the complexity of the calculations. Grim (1953) justified his theoretical damping curves by an analysis of coupled pitching and heaving oscillations of several ship models. The oscillations were induced by means of rotating unbalanced masses; i.e., with a known value of the exciting function. Korvin-Kroukovsky and Jacobs (1957) successfully used Grim's damping coefficients in the analysis of several ship models which had been tested in towing tanks. Korvin-Kroukovsky and Lewis (1955) and Korvin-Kroukovsky (1955c) previously made similar use of Havelock's coefficients.

Direct measurements of damping on a ship model were made by Golovato (1957a and b) and Gerritsma (1957c and d, 1958, 1960).¹⁷ The ship model used by Golovato was a

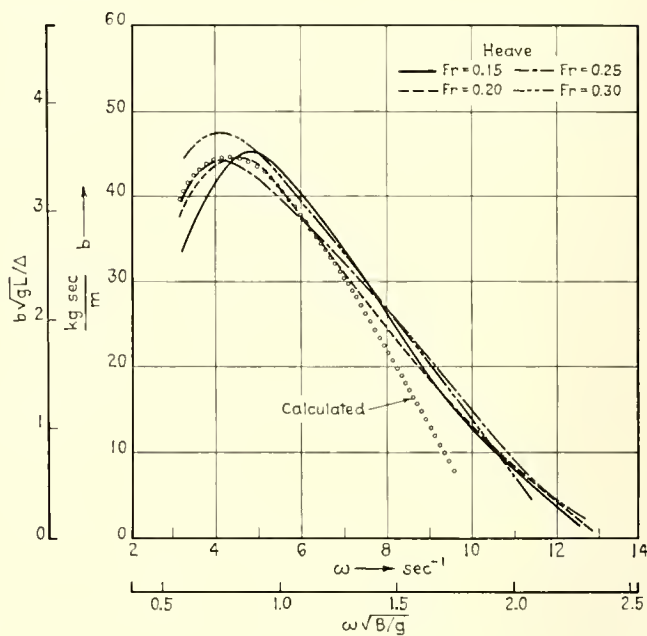


Fig. 12 Comparison of measured damping in heave with one calculated by strip theory using Holstein-Havelock method (from Gerritsma, 1957c)

mathematically defined form, symmetrical fore and aft. Various hydrodynamic forces were measured experimentally in a simple heaving motion (pitching restrained) induced by a mechanical oscillator. The results for damping are shown in Fig. 14 which also includes damping as computed by Havelock's and Grim's methods. The general trend of the damping-force variations as given by all three methods is identical, but both calculated methods give higher damping than the experimental values. (In connection with ship motion analysis, it is suggested that the reader concentrate his attention on the abscissa range 0.9 to 1.5.)

Fig. 7, taken from Gerritsma (1957d), shows a comparison of damping-force coefficients measured on a Series 60, 0.60 block coefficient model with those computed by Korvin-Kroukovsky and Jacobs (1957) by means of the strip theory, using Grim's (1953) material. Figs. 12 and 13 show a similar comparison with the damping computed by Korvin-Kroukovsky (1955c) using the Holstein-Havelock method.

In comparing the heave damping data of Golovato and Gerritsma, it is observed that the measured damping of the Series 60 model is much higher than that of the idealized model: the maximum nondimensional value in Fig. 12 is about 3.7 as against 2 in Fig. 14. A part of this drastic increase can be explained by the presence of inclined ship sides in the afterbody of the Series 60 model, while Golovato's idealized model was wall-sided.

The theoretically evaluated damping is also higher for the Series 60 model, but the increase is not as drastic as for the measured damping. This difference may result from the fact that the inclination of ship sides at the

¹⁶ Dimpker's (1934) paper is a part of a Göttingen dissertation prepared under the guidance of M. Schuler and L. Prandtl.

¹⁷ Added mass data from these tests were discussed in Section 3.15-3.

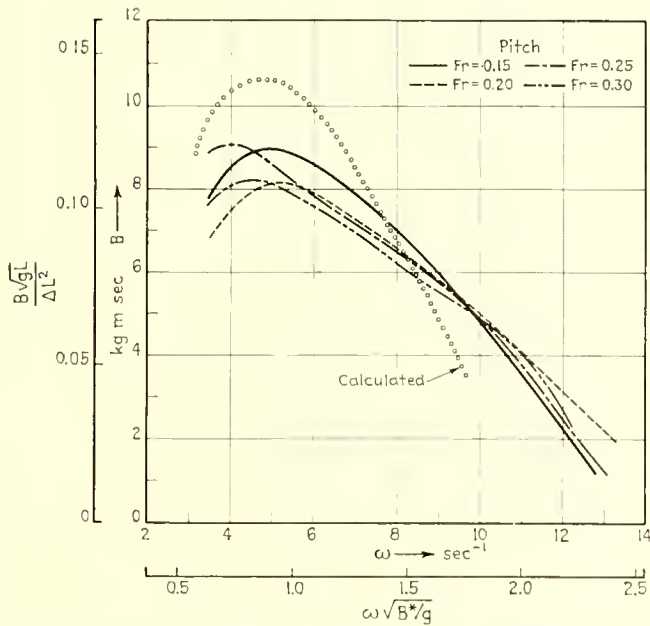


Fig. 13 Comparison of measured damping in pitch with one calculated by strip theory using Holstein-Havelock method (from Gerritsma, 1957c)

waterline is not taken into consideration in the available methods of calculation.

In Golovato's tests the calculated damping greatly exceeded the measured damping, and Grim's theoretical method gave the better approximation. With the different rates of increase of measured and calculated damping for a practical ship form, Gerritsma's heaving test indicates an apparently excellent agreement with the Holstein-Havelock method. Fig. 7 shows that Grim's method underestimated the damping in heave. The word "apparently" was used advisedly. Were the increase of damping due to inclined ship sides taken into account in calculations, both curves of theoretical damping in Figs. 7 and 12 would be displaced upwards.

Examination of Figs. 7 and 13 indicates that the relationship between calculated and measured damping in pitching is drastically different from that in heaving. In the case of pitching, Grim's method is found to agree with the measured data, while the Holstein-Havelock method exaggerates the damping. The calculated damping would be further increased if ship side inclinations were taken into account.

The shift from agreement to disagreement of the calculated and measured damping in the cases of heaving and pitching oscillations indicates a strong three-dimensional effect. However, application of the three-dimensional corrections developed by Havelock and Vossers (to be discussed in Section 3.23) would make the situation still worse. The pitch-damping curves (at synchronous frequency) would be displaced upwards, while the heave-damping curves would not be affected.

To summarize: Prediction of the damping of a ship's

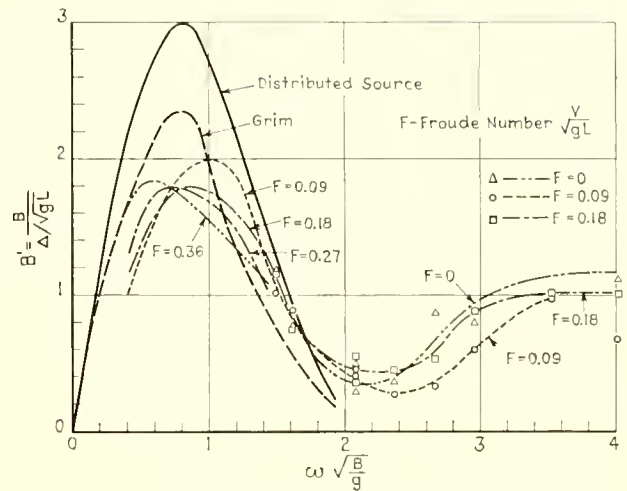


Fig. 14 Variation of damping coefficient with frequency (from Golovato, 1957)

heaving and pitching motions is rather uncertain. The order of magnitude and the functional dependence of the damping on oscillation frequency can be estimated roughly. Neither of the two available methods of calculating, Grim's or Holstein-Havelock's, gives uniformly satisfactory results. Fortunately, one or the other will be preferable in a particular case. Currently available calculations of three-dimensional effects (Section 3.23) do not correct discrepancies but apparently make the situation worse. The most pressing need in the theory of ship motions and ship bending stresses is to develop a reliable method of evaluating the damping characteristics.

Experimental measurements on idealized ship forms may be misleading if used directly as an indication of normal ship behavior. Tests on such models are, however, recommended, but only for comparison with calculated values since more advanced methods of calculation can be used for such mathematically defined ship forms than is possible for normal ship forms (Section 6). It would be desirable to develop mathematical ship lines which would be more like the normal ship form and yet permit application of the advanced calculation methods.

The erratic correlation between computed and measured damping when comparison is made of mathematical and normal ship forms, and of pitching and heaving motions, indicates that the phenomenon is caused by a complexity of conditions. A research program should be directed therefore towards resolving this complex phenomenon into its (not now known) parts and towards subsequent analysis of these component parts. Two general directions of approach can be visualized. In one, tests similar to Golovato's and Gerritsma's would be conducted on a variety of ship forms. Various factors can then be isolated intuitively after inspection of the test data, and the conclusions can be verified subsequently by synthesis of the elemental findings. The other

INTRODUCTION IN SHIP HYDROMECHANICS

Lecture MT519

Draft Edition



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Ship Motions and Sea Loads

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A new strip theory is presented for predicting heave, pitch, sway, roll, and yaw motions as well as wave-induced vertical and horizontal shear forces, bending moments, and torsional moments for a ship advancing at constant speed with arbitrary heading in regular waves. A computer program based on this theory and with accurate close-fit section representation has been developed. Comparisons between computed values and experimental data show satisfactory agreement in general. In particular, very good agreement is shown for the heave and pitch motions and the vertical loads. Accurate results are also obtained for the coupled sway-roll motions in beam waves. Although comparisons are not yet available for the sway-roll-yaw motions in oblique waves, the satisfactory agreement shown for the horizontal loads in oblique waves suggests that the theory may also predict the horizontal motions quite well.

1. Introduction

Preface

THE ULTIMATE criterion for the hull design of a ship should be the performance of the ship in a realistic seaway. Prediction of the ship motions and the dynamic sea loads is such a complex problem, however, that the naval architect has been forced to use the ship's effective power performance in calm water and the ship's maximum bending moment in the static "one-over-twenty" wave as his main design criteria. Until very recently ship motions and wave-induced loads were barely considered in the design procedure.

The design of high-speed dry-cargo ships and huge tankers has made us more aware of the importance of reducing the ship motions and of mini-

mizing the wave-induced loads. Considering the importance of the seaworthiness problem, it is very encouraging indeed to note the tremendous advancement in this field over the past two decades.

The well-known paper of St. Denis and Pierson (1953)⁴ on the application of the principle of superposition to the ship-motion problem started a new era in this field by hypothesizing that the responses of a ship to irregular waves can be considered as the summation of the responses to regular waves of all frequencies. Today the validity of the application of the superposition to ship motion and sea loads is generally accepted in our field, and in particular for the vertical motions and loads this validity "may be considered as proven, beyond the fondest hopes of earlier investigators" (Ogilvie, 1964). Assuming that the principle of superposition is also valid for the horizontal responses, the complex problem of predicting ship motions and sea loads in a seaway can be reduced to the two problems: (i) the prediction of the ship motions and loads in regular

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⁴ References are listed in alphabetical order at the end. In the paper itself they are identified by author's name and year of publication.

sinusoidal waves and (ii) the prediction of the statistical responses in irregular waves using the regular wave results.

If the responses for a ship in regular waves are known, there are now available procedures which follow the method of St. Denis and Pierson for determining the statistical responses not only for a given sea state, but for a distribution of sea conditions which a ship may encounter in its life span (Abrahamsen, 1967). However, a major difficulty in seaworthiness analysis has been to make accurate predictions of motions and sea loads for a ship in regular waves. Therefore the objective of this paper is to present a practical numerical method with sufficient engineering accuracy for predicting the heave, pitch, sway, roll, and yaw motions as well as the wave-induced shear forces, bending moments, and torsional moments for a ship advancing at constant speed at arbitrary heading in regular sinusoidal waves.

With the motion and load theory presented here and with the available statistical methods, it is felt that the naval architect will have a useful tool for determining the seaworthiness characteristics of a ship. If the designer knows the geometric description and the weight distribution and has adequate information about the sea environment, he can calculate the motions and the dynamic loads for a ship in a seaway with reasonable accuracy.

Historical Background

Since the St. Denis and Pierson paper, there have been spectacular developments in both experimental and theoretical methods for predicting ship responses in regular waves. Large experimental facilities for testing models in oblique waves were in full operation in 1956 at the Netherlands Ship Model Basin and a year later at the Davidson Laboratory, and during the next ten years such facilities were built at the Naval Ship Research and Development Center, the Admiralty Experimental Works in Haslar, England, and at the Ship Research Institute in Mitaka, Tokyo.⁵ Furthermore, most of the tanks originally designed for resistance and propulsion tests have been equipped with wavemakers so that they can be used for head- and following-wave experiments. Numerous ship-motion and wave-load tests have been conducted in these facilities, but perhaps the most significant and comprehensive tests are the systematic experiments conducted at NSMB in Wageningen on sixteen different Series 60 hull

⁵ Very recently a smaller seakeeping laboratory was completed at the University of Tokyo.

forms. The motions, the power increase, and the wave-induced loads were measured for each hull in head, following, and oblique regular waves (Vossers, Swaan, Rijken, 1960 and 1961). These data have been invaluable in the study of the hull-form effect on seakeeping characteristics. Unfortunately, for hull forms not closely related to the Series 60 forms there exist no similar systematic experimental data. In fact for the non-Series 60 forms most of the published data have been only for heave and pitch motions in head seas.

Since ship-motion and sea-load experiments are extremely expensive and time consuming, it is not usually feasible to perform these experiments for individual ship designs. Therefore the paper of St. Denis and Pierson has further emphasized the importance of the development of theoretical and numerical methods for predicting the regular wave responses. The strip theory for heave and pitch motions in head waves of Korvin-Kroukovsky and Jacobs (1957) was the first motion theory suitable for numerical computations which had adequate accuracy for engineering applications. This theory was later extended by Jacobs (1958) to include the wave-induced vertical shear forces and bending moments for a ship in regular head waves.

It is now apparent that the theory of Korvin-Kroukovsky and Jacobs did not receive the recognition it deserved. Purists felt that the theory was not derived in a rational mathematical manner but rather by use of "physical intuition." Today, however, after more sophisticated motion theories have been derived and more accurate experimental data are available, it is becoming clear that this original strip theory is one of the most significant contributions in the field of seakeeping. It has been demonstrated in numerous publications over the past ten years that the theory predicts the heave and pitch motions as well as the vertical shear forces and bending moments with amazing accuracy for regular cruiser stern ships at moderate speeds in head waves.

The Korvin-Kroukovsky and Jacobs theory has since been modified and extended. For example, W. E. Smith (1967) has shown that a modified strip theory by Gerritsma and Beukelman (1967) predicts the head-seas motions for a high-speed destroyer hull which agree quite well with experiments. In particular, by the use of close-fit methods, very significant improvements have been made in the computation of the sectional added-mass and damping coefficients, and Smith and Salvesen (1970) have demonstrated that the head-seas motions can be predicted quite accurately even for high-speed hulls with large bulbous bows

when such close-fit methods are applied. It should also be noted that attempts have been made to extend the original head-seas strip theory to the case of heave and pitch in oblique seas; however, these extended oblique-seas theories are not that accurate since the diffraction effect in the exciting force has not been treated properly.

Even though the agreement between experiments and the Korvin-Kroukovsky and Jacobs strip theory has usually been quite satisfactory, a major objection to this theory has been that the forward-speed terms in the coefficients of the equations of motion do not satisfy the symmetry relationship proved by Timman and Newman (1962). During the past year, however, new strip theories for heave and pitch motions in head waves have been derived independently in Germany by Söding (1969), in Japan by Tasai and Takaki (1969), and in the Soviet Union by Borodai and Netsvetayev (1969). All of these new strip theories have identical forward-speed terms satisfying the Timman and Newman symmetry relationships, and, interestingly enough, the equations of motion for heave and pitch in head waves derived in the present work have the same speed terms as those given in these three recent publications.

It should be mentioned that Ogilvie and Tuck (1969) have derived a completely new strip theory for head seas by using slender-body theory. Unfortunately, there are some integral terms in their theory which have not yet been evaluated; thus their theory cannot be fully utilized or judged at this time.

For the sway, yaw, and roll motions and for the horizontal wave-induced loads, there exist few computational methods. Tasai (1967) has derived a strip theory for the sway-yaw-roll motions, but this theory is only applicable for the case of zero forward speed. Grim and Schenzle (1969), on the other hand, have considered forward-speed effects in their strip theory, which does include the sway-yaw-roll motions as well as the horizontal loads. However, the forward-speed terms in their equations of motion do not satisfy the Timman and Newman (1962) symmetry relationships and their theory lacks many of the forward-speed terms included in the theory presented herein. Furthermore, comparisons between experiments and the theory of Grim and Schenzle exist only for the case of zero forward speed.

Present Theory

The theory presented herein can predict the heave, pitch, sway, roll, and yaw motions as well as the wave-induced vertical and horizontal shear

forces, bending moments, and torsional moments for a ship advancing at constant speed in regular waves.

Only the final equations are stated in the main text while a detailed derivation of the hydrodynamic coefficients is presented in the Appendices. The derived equations of motion consist of two sets of linear coupled differential equations with frequency- and speed-dependent coefficients. One set of equations is for the heave-pitch motions and the other is set for the sway-yaw-roll motions. The equations for the wave-induced loads are expressed in terms of the resulting motions and the derived hydrodynamic coefficients.

A computer program based on this theory has been developed jointly by the Naval Ship Research and Development Center, Washington, D. C. and Det norske Veritas, Oslo, Norway. The ship-motion part of the program was originally written by Werner Frank at the NSRDC. Frank (1967) also developed the close-fit source-distribution technique used in the program for computing the two-dimensional added-mass and damping coefficients. The program was later improved and extended at Det norske Veritas to include the wave-induced loads. All the numerical results presented here have been computed by this program on the Univac 1108 at Det norske Veritas. A documentation of the program including a users manual and a program listing will soon be available as an NSRDC Report.

Comparisons between computed values and experimental data are also presented. The agreement is very satisfactory for the heave and pitch motions and the vertical loads in oblique and following waves as well as in head waves. Good agreement between theory and experiments is also obtained for the coupled sway-roll motions in beam waves, while owing to lack of experimental data it has not been possible to make comparisons for the sway-roll-yaw motions in oblique waves. Nevertheless, the good agreement shown for the horizontal shear forces, bending moments, and torsional moments in oblique waves suggests that the theory may also predict the horizontal motions quite well.

2. Ship Motions

The equations of motion are presented in this section for a ship advancing at constant mean forward speed with arbitrary heading in regular sinusoidal waves. The equations for pitch and heave motions in head waves are compared with the original strip theory of Korvin-Kroukovsky and Jacobs (1957). Comparisons between computed and experimental motion values are also shown.

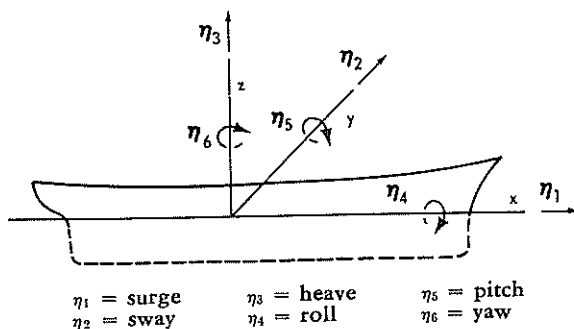


Fig. 1 Sign convention for translatory and angular displacements

General Formulation of Equations of Motion

It is assumed that the oscillatory motions are linear and harmonic. Let (x, y, z) be a right-handed coordinate system fixed with respect to the mean position of the ship with z vertically upward through the center of gravity of the ship, x in the direction of forward motion, and the origin in the plane of the undisturbed free surface. Let the translatory displacements in the x , y , and z directions with respect to the origin be η_1 , η_2 , and η_3 , respectively, so that η_1 is the surge, η_2 is the sway, and η_3 is the heave displacement. Furthermore, let the angular displacement of the rotational motion about the x , y , and z axes be η_4 , η_5 , and η_6 , respectively, so that η_4 is the roll, η_5 is the pitch, and η_6 is the yaw angle. The coordinate system and the translatory and angular displacements are shown in Fig. 1.

Under the assumptions that the responses are linear and harmonic, the six linear coupled differential equations of motion can be written, using subscript notation, in the following abbreviated form:

$$\sum_{k=1}^6 [(M_{jk} + A_{jk})\ddot{\eta}_k + B_{jk}\dot{\eta}_k + C_{jk}\eta_k] = F_j e^{i\omega t}; \quad j = 1 \dots 6 \quad (1)$$

where M_{jk} are the components of the generalized mass matrix for the ship, A_{jk} and B_{jk} are the added-mass and damping coefficients,⁶ C_{jk} are the hydrostatic restoring coefficients,⁷ and F_j are

⁶ Note that A_{jk} (for $j \neq k$) are the added-mass cross-coupling coefficients for the k th mode coupled into the j th mode of motion, so that for example A_{35} is the added-mass coefficient for pitch coupled into heave.

⁷ Here C_{jk} are defined as the hydrostatic restoring coefficients and hence independent of frequency, while the added-mass coefficients A_{jk} are so defined that they include all the oscillatory hydrodynamic forces proportional to the acceleration. Some other authors prefer to include certain hydrodynamic terms in the C_{jk} 's which are included in the A_{jk} 's here.

the complex amplitudes of the exciting force and moment, with the force and moment given by the real part of $F_j e^{i\omega t}$.⁸ F_1 , F_2 , and F_3 refer to the amplitudes of the surge, sway, and heave exciting forces, while F_4 , F_5 , and F_6 are the amplitudes of the roll, pitch, and yaw exciting moments; ω is the frequency of encounter and is the same as the frequency of the response. The dots stand for time derivatives so that $\dot{\eta}_k$ and $\ddot{\eta}_k$ are velocity and acceleration terms.

If it is assumed that the ship has lateral symmetry (symmetric about the x, z plane) and that the center of gravity is located at $(0, 0, z_c)$, then the generalized mass matrix is given by

$$M_{jk} = \begin{bmatrix} M & 0 & 0 & 0 & Mz_c & 0 \\ 0 & M & 0 & -Mz_c & 0 & 0 \\ 0 & 0 & M & 0 & 0 & 0 \\ 0 & -Mz_c & 0 & I_4 & 0 & -I_{46} \\ Mz_c & 0 & 0 & 0 & I_5 & 0 \\ 0 & 0 & 0 & -I_{46} & 0 & I_6 \end{bmatrix} \quad (2)$$

where M is the mass of the ship, I_j is the moment of inertia in the j th mode, and I_{jk} is the product of inertia. Here the inertia terms are with respect to the coordinate system shown in Fig. 1. The only product of inertia which appears is I_{46} , the roll-yaw product, which vanishes if the ship has fore-and-aft symmetry and is small otherwise. The other nondiagonal elements all vanish if the origin of the coordinate system coincides with the center of gravity of the ship; however, it is frequently more convenient to take the origin in the water plane, in which case z_c is not equal to zero.

For ships with lateral symmetry it also follows that the added-mass (or damping) coefficients are

$$A_{jk} \text{ (or } B_{jk}) = \begin{bmatrix} A_{11} & 0 & A_{13} & 0 & A_{15} & 0 \\ 0 & A_{22} & 0 & A_{24} & 0 & A_{26} \\ A_{31} & 0 & A_{33} & 0 & A_{35} & 0 \\ 0 & A_{42} & 0 & A_{44} & 0 & A_{46} \\ A_{51} & 0 & A_{53} & 0 & A_{55} & 0 \\ 0 & A_{62} & 0 & A_{64} & 0 & A_{66} \end{bmatrix} \quad (3)$$

Furthermore, for a ship in the free surface the only nonzero linear hydrostatic restoring coefficients are

$$C_{33}, C_{44}, C_{55}, \text{ and } C_{35} = C_{53} \quad (4)$$

If the generalized mass matrix (2), the added-mass and damping coefficients (3), and the restoring coefficients (4) are substituted in the equations of motions (1), it is seen that for a ship with lateral symmetry, the six coupled equations of motions (1) reduce to two sets of equations: one set of

⁸ It is understood that real part is to be taken in all expressions involving $e^{i\omega t}$.

three coupled equations for surge, heave, and pitch and another set of three coupled equations for sway, roll, and yaw. Thus, for a ship with lateral symmetry, surge, heave, and pitch are not coupled with sway, roll, and yaw.

If one assumes that the ship has a long slender hull form in addition to lateral symmetry, then it can be shown (as seen in Appendix 1) that the hydrodynamic forces associated with the surge motion are much smaller than the forces associated with the five other modes of motion so that it is consistent within these assumptions not to include surge. Hence the three coupled equations of motion for surge, heave, and pitch reduce to two coupled equations for pitch and heave.

Heave and Pitch Motions

Under the assumption that the oscillatory motions are linear and harmonic, it follows from equations (1) through (4) that for a ship with lateral symmetry and a slender hull form the

coupled equations of motion for heave and pitch can be written in the form

$$(M + A_{33})\ddot{\eta}_3 + B_{33}\dot{\eta}_3 + C_{33}\eta_3 + A_{35}\ddot{\eta}_5 + B_{35}\dot{\eta}_5 + C_{35}\eta_5 = F_3 e^{i\omega t} \quad (5)$$

$$A_{53}\ddot{\eta}_3 + B_{53}\dot{\eta}_3 + C_{53}\eta_3 + (I_5 + A_{55})\ddot{\eta}_5 + B_{55}\dot{\eta}_5 + C_{55}\eta_5 = F_5 e^{i\omega t} \quad (6)$$

The relationships for the added-mass and damping coefficients, A_{jk} and B_{jk} , and the amplitude of the exciting force and moment, F_3 and F_5 , are derived in Appendix 1. However, the main assumptions made in the derivation in Appendix 1 are significant in the application of the theory and therefore will be restated here. First of all it is assumed that all viscous effects can be disregarded. Hence, the only damping considered is the damping due to the energy loss in creating free-surface waves. This assumption is justified because the viscous damping is very small for the vertical ship motions. Furthermore, in order to linearize the

Nomenclature

(Additional nomenclature used in the Appendices are defined only as they appear)

A_{jk} = added-mass coefficients ($j, k = 1, 2 \dots 6$)	b_{jk} = two-dimensional sectional damping coefficient
A_{jk}^0 = speed-independent part of A_{jk}	b_{jk}^A = b_{jk} for aftermost section
A_{WP} = area of water plane	b_{44}^* = sectional viscous damping in roll
B = ship beam	d = sectional draft
B_{jk} = damping coefficients	dl = element of arc along a cross section
B_{jk}^0 = speed-independent part of B_{jk}	f_j = sectional Froude-Kriloff "force"
B_{44}^* = viscous damping in roll	g = gravitational acceleration
C_{jk} = hydrostatic restoring coefficients	h_j = sectional diffraction "force"
C_x = cross section at x	h_j^A = h_j for aftermost section
D_j = hydrodynamic force and moment due to body motion	i_x = sectional mass moment of inertia about x -axis
E_j = exciting force and moment on portion of hull	j, k = subscripts ($j, k = 1, 2 \dots 6$)
F_j = exciting force and moment	k = wave number
F_n = Froude number	m = sectional mass per unit length
\overline{GM} = metacentric height	\overline{om} = sectional metacentric height
I_j = moment of inertia in j th mode	s = sectional area coefficient
I_{jk} = product of inertia	t = time variable
I_{WP} = moment of inertia of water plane	x, y, z = coordinate system as defined in Fig. 1
K = damping coefficient	x_A = x -coordinate of aftermost cross section
L = length between perpendiculars	z_c = z -coordinate of center of gravity
M = mass of ship	\bar{z} = z -coordinate of sectional center of gravity
M_{jk} = generalized mass matrix for ship	∇ = displaced volume of ship
M_{WP} = moment of water plane	α = incident wave amplitude
N_j = two-dimensional sectional generalized normal components ($j = 2, 3, 4$)	β = angle between incident wave and ship heading ($\beta = 180$ deg for head seas); see Fig. 2
R_j = restoring force on portion of hull	η_j = displacements, ($j = 1, 2 \dots 6$ refer to surge, sway, heave, roll, pitch, and yaw respectively; see Fig. 1)
U = ship speed	λ = wave length
V_j = dynamic load components (see Fig. 9 for definitions)	ξ = variable of integration in x -direction
a = submerged sectional area	ρ = mass density of water
a_{jk} = two-dimensional sectional added-mass coefficient	ψ_j = two-dimensional velocity potential
a_{jk}^A = a_{jk} for aftermost section	ω = frequency of encounter
b = sectional ship beam	ω_0 = wave frequency

potential problem, it is assumed that the wave-resistance perturbation potential and all its derivatives are small enough to be ignored in the formulation of the motion problem.⁹ Physically this means that the free-surface waves created by the ship advancing at constant speed in calm water are assumed to have no effect on the motions. This appears to be a reasonable assumption for fine slender hull forms.

Finally, in order to reduce the three-dimensional problem to a summation of two-dimensional problems, it is necessary to assume that the frequency is (relatively) high. This means that the waves created by the ship's oscillations should have a wave length of the order of the ship beam rather than the ship length. This is a critical assumption since the maximum responses are in the fairly low-frequency range (the long-wave range); however, the pitch and heave motions in the low-frequency range are dominated by the hydrostatic forces so that inaccuracies in the hydrodynamic coefficients in this range have a minor effect on the final results.

The added-mass and damping coefficients as derived in Appendix 1 are

$$A_{33} = \int a_{33} d\xi - \frac{U}{\omega^2} b_{33}^A \quad (7)$$

$$B_{33} = \int b_{33} d\xi + U a_{33}^A \quad (8)$$

$$A_{35} = - \int \xi a_{33} d\xi - \frac{U}{\omega^2} B_{33}^0 + \frac{U}{\omega^2} x_A b_{33}^A - \frac{U^2}{\omega^2} a_{33}^A \quad (9)$$

$$B_{35} = - \int \xi b_{33} d\xi + U A_{33}^0 - U x_A a_{33}^A - \frac{U^2}{\omega^2} b_{33}^A \quad (10)$$

$$A_{53} = - \int \xi a_{33} d\xi + \frac{U}{\omega^2} B_{33}^0 + \frac{U}{\omega^2} x_A b_{33}^A \quad (11)$$

$$B_{53} = - \int \xi b_{33} d\xi - U A_{33}^0 - U x_A a_{33}^A \quad (12)$$

$$A_{55} = \int \xi^2 a_{33} d\xi + \frac{U^2}{\omega^2} A_{33}^0 - \frac{U}{\omega^2} x_A^2 b_{33}^A + \frac{U^2}{\omega^2} x_A a_{33}^A \quad (13)$$

$$B_{55} = \int \xi^2 b_{33} d\xi + \frac{U^2}{\omega^2} B_{33}^0 + U x_A^2 a_{33}^A + \frac{U^2}{\omega^2} x_A b_{33}^A \quad (14)$$

Here a_{33} and b_{33} are the two-dimensional sectional added-mass and damping coefficients for heave. All the integrals are over the length of the ship and U is the forward speed of the ship. A_{33}^0 and B_{33}^0 refer to the speed-independent part of A_{33} and B_{33} ; x_A is the x -coordinate of the aftermost cross-section of the ship; and a_{33}^A and b_{33}^A are the added-mass and damping coefficients for the aftermost section.

The hydrostatic restoring coefficients, which are independent of frequency and forward speed, follow directly from hydrostatic considerations as

$$C_{33} = \rho g b d \xi = \rho g A_{WP} \quad (15)$$

$$C_{35} = C_{53} = - \rho g \int \xi b d \xi = - \rho g M_{WP} \quad (16)$$

$$C_{55} = \rho g \int \xi^2 b d \xi = \rho g I_{WP} \quad (17)$$

Here b is the sectional beam of the ship, ρ is the mass density of the water, g is the gravitational acceleration, and the integration is over the length of the ship. A_{WP} , M_{WP} , and I_{WP} are the area, moment, and moment of inertia of the water plane.

The amplitudes of the exciting force and moment as derived in Appendix 1 are

$$F_3 = \rho \alpha \int (f_3 + h_3) d\xi + \rho \alpha \frac{U}{i\omega} h_3^A \quad (18)$$

$$F_5 = - \rho \alpha \int \left[\xi (f_3 + h_3) + \frac{U}{i\omega} h_3 \right] d\xi - \rho \alpha \frac{U}{i\omega} x_A h_3^A \quad (19)$$

with the sectional Froude-Kriloff "force" defined by

$$f_3(x) = g e^{-ikx \cos \beta} \int_{C_x} N_3 e^{iky \sin \beta} e^{kz} dl \quad (20)$$

and the sectional diffraction "force" by

$$h_3(x) = \omega_0 e^{-ikx \cos \beta} \int_{C_x} (iN_3 - N_2 \times \sin \beta) e^{iky \sin \beta} e^{kz} \psi_3 dl \quad (21)$$

Here α is the wave amplitude, k is the wave number, β is the heading angle (see Fig. 2 for definitions), dl is an element of arc along the cross section C_x , and $\omega_0 = \sqrt{gk}$ is the wave frequency which is related to ω , the frequency of encounter, by

$$\omega_0 = \omega + kU \cos \beta \quad (22)$$

Furthermore, h_3^A refers to h_3 for the aftermost sec-

⁹ But it must be emphasized that this is on a prior assumption of the present theory. For example, the analysis of Ogilvie and Tuck (1969) includes some contributions (believed to be small) arising from interaction between the steady and unsteady flow fields.

$$G_j^* = \rho \sum_{k=1}^6 \zeta_k \left\{ -i\omega \iint_{S^*} n_j^* \phi_k ds + U \iint_{S^*} m_j \phi_k ds - U \int_{C_x} n_j^* \phi_k dl \right\} \quad (155)$$

Now setting $ds = dld\xi$, we have

$$G_j^* = \rho \sum_{k=1}^6 \zeta_k \left\{ -i\omega \int_{L^*} \int_{C_x} n_j^* \phi_k dld\xi + U \int_{L^*} \int_{C_x} m_j \phi_k dld\xi - U \int_{C_x} n_j^* \phi_k dl \right\} \quad (156)$$

Here L^* is the length of the hull forward of cross section C_x . If the "strip-theory" assumptions are introduced, as in Appendix 1, it follows that the six components of the generalized three-dimensional normal can be expressed in terms of the two-dimensional general N_j in the form

$$n_j^* = (0, N_2, N_3, N_4, -(\xi - x)N_3, (\xi - x)N_2) \quad (157)$$

and the velocity potential at a given section can be expressed in terms of the two-dimensional potential, ψ_k ($k = 2, 3, 4$), as

$$\phi_1 \approx 0 \text{ and } \phi_k = \psi_k; \quad k = 2, 3, 4$$

$$\phi_5 = -\xi\psi_3 + \frac{U}{i\omega} \psi_3 \quad (158)$$

$$\phi_6 = \xi\psi_2 - \frac{U}{i\omega} \psi_2$$

Use of equations (157) and (158) in equation (156) enables the force and moment amplitudes to be expressed in terms of the sectional line integral

$$t_{jk} = -\rho i\omega \int_C N_j \psi_k dl; \quad j, k = 2, 3, 4 \quad (159)$$

The force amplitude components are

$$G_2^* = \int_{L^*} \left\{ \left(\zeta_2 + \xi\zeta_6 - \frac{U}{i\omega} \zeta_6 \right) t_{22} + \zeta_4 \zeta_{24} \right\} d\xi + \frac{U}{i\omega} \left[\left(\zeta_2 + \xi\zeta_6 - \frac{U}{i\omega} \zeta_6 \right) t_{22} + \zeta_4 t_{24} \right]_{\xi=x} \quad (160)$$

$$G_3^* = \int_{L^*} \left(\zeta_3 - \xi\zeta_5 + \frac{U}{i\omega} \zeta_5 \right) t_{33} d\xi + \frac{U}{i\omega} \left[\left(\zeta_3 - \xi\zeta_5 + \frac{U}{i\omega} \zeta_5 \right) t_{33} \right]_{\xi=x} \quad (161)$$

and the moment amplitude components are

$$G_4^* = \int_{L^*} \left\{ \left(\zeta_2 + \xi\zeta_6 - \frac{U}{i\omega} \zeta_6 \right) t_{24} + \zeta_4 t_{44} \right\} d\xi + \frac{U}{i\omega} \left[\left(\zeta_2 + \xi\zeta_6 - \frac{U}{i\omega} \zeta_6 \right) t_{24} + \zeta_4 t_{44} \right]_{\xi=x} \quad (162)$$

$$G_5^* = - \int_{L^*} (\xi - x)(\zeta_3 - \xi\zeta_5) t_{33} d\xi + \frac{U}{i\omega} \left(-\zeta_3 + x\zeta_5 - \frac{U}{i\omega} \zeta_5 \right) \int_{L^*} t_{33} d\xi \quad (163)$$

$$G_6^* = \int_{L^*} \{ (\xi - x)(\zeta_2 + \xi\zeta_6) t_{22} + (\xi - x)\zeta_4 t_{24} \} d\xi + \frac{U}{i\omega} \left(\zeta_2 + x\zeta_6 - \frac{U}{i\omega} \zeta_6 \right) \int_{L^*} t_{22} d\xi + \frac{U}{i\omega} \zeta_4 \int_{L^*} t_{24} d\xi \quad (164)$$

One may go one step further and express these force and moment components in terms of real variables. If we let

$$D_j = \text{Re} G_j^* e^{i\omega t} \text{ and } \eta_j = \text{Re} \zeta_j e^{i\omega t} \quad (165)$$

and use $\omega^2 a_{jk} - i\omega b_{jk} = t_{jk}$, the hydrodynamic force and moment due to the body motion are those presently in the main text of the paper, equations (78) through (82), in terms of the velocity and acceleration, $\dot{\eta}_j$ and $\ddot{\eta}_j$, and the sectional added-mass and damping coefficients, a_{jk} and b_{jk} .